The Tower Problem

You have been hired by a contractor to design buildings with towers. The contractor has some thoughts about these buildings. She knows that it will take 6 cubes to build a building with 1 cube as its tower. It takes 7 cubes to build a building with 2 cubes for its tower. It takes 8 cubes to build a building with 3 cubes as its tower.

There are 2 questions the contractor would like you to explain to her. First, how many cubes are in a building that has a tower that is 10 cubes high? Secondly, can you find a rule that will allow the contractor to find the number of cubes in a building with any size tower?

Be sure to explain or show your reasoning to the contractor.

Grade Levels 3 - 5

The Tower Problem

You have been hired by a contractor to design buildings with towers. The contractor has some thoughts about these buildings. She knows that it will take 6 cubes to build a building with 1 cube as its tower. It takes 7 cubes to build a building with 2 cubes for its tower. It takes 8 cubes to build a building with 3 cubes as its tower.

There are 2 questions the contractor would like you to explain to her. First, how many cubes are in a building that has a tower that is 10 cubes high? Secondly, can you find a rule that will allow the contractor to find the number of cubes in a building with any size tower?

Be sure to explain or show your reasoning to the contractor.

Context

We are always looking at patterns in class. This problem allows students to see patterns not just in numbers, but also patterns using manipulatives. The more children can relate equations to situations, the more comfortable most students will be in understanding the meaning of operations.

What This Task Accomplishes

This assessment allows students to see that patterns can help to solve problems. It also shows that making a chart to find a pattern is useful. It pushes students to try to come up with a generalization - something that has traditionally been left to the high school algebra class.

What the Student Will Do

Most students will experiment with different bases for their buildings. Once that is determined then they usually build the three buildings that were described and try to build the next few buildings. At this point they begin to have a need to organize their information. Many students will use a chart. Most students can find the number of cubes for a building with a tower 10 cubes high, not all students will be able to come to a generalization.

Time Required for Task

45 minutes

Interdisciplinary Links

This task can be given in conjunction with a unit on architecture.

Teaching Tips

The time for this task depends on how familiar the students are with working with cubes. You may want to take the cubes out the day before and give students time to play with them.

Suggested Materials

- Cubes (for constructing buildings)
- Paper
- Pencil

Possible Solutions

It will take 15 cubes to build a building with a 10-cube tower. The rule can be stated in words like, "It always will take 5 more cubes than the number of cubes in the tower" or in equation form such as: T (number of cubes in tower) + 5 = N (number of cubes all together). Students may express their generalizations in many different ways.

Benchmark Descriptors

Novice
There does not seem to be any strategy or evidence of mathematical reasoning. The explanation of the solution does not seem to relate to the problem.

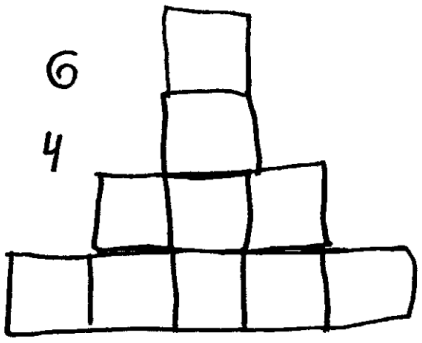
Apprentice
The student uses a strategy that is partially successful, leading to part of the solution, but that strategy does not help in coming up with a correct generalization. There is some use of correct mathematical representation.

Practitioner
The solution shows that the student has a broad understanding of the major concepts necessary for the solution. A strategy is used that leads to a solution of the problem. There is appropriate use of accurate mathematical representation and mathematical terminology.

Expert
The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts. The student has an efficient equation for the generalization. There is accurate and appropriate use of mathematical representations and terminology.

Novice

Cubs in tower	Cubs in all
1	6
2	7
3	8
4	6
5	4



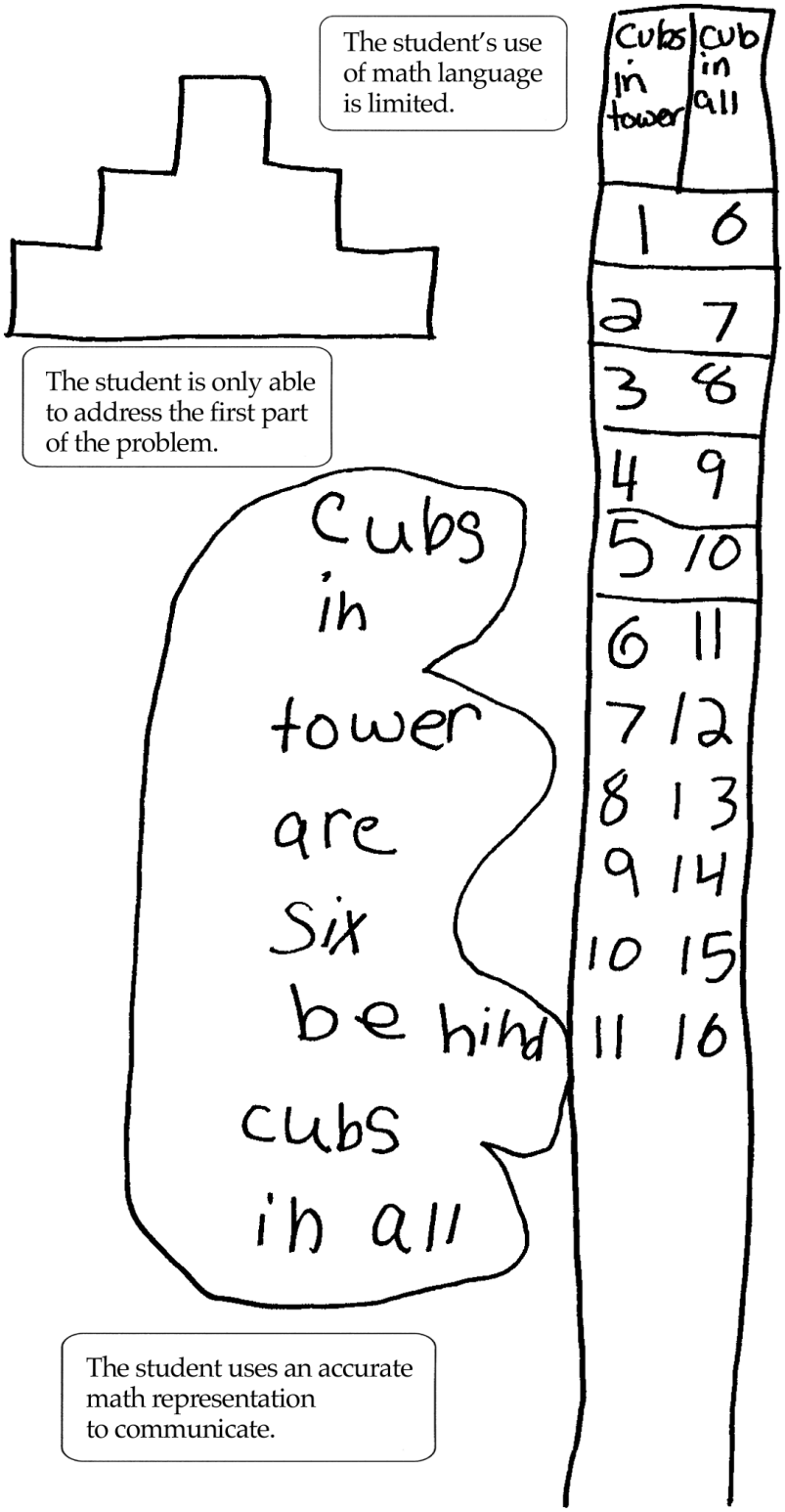
The student is unable to find a pattern and an approach that would work.

The student attempts to make a relevant observation although the student's communication of that observation is unclear.

We are unclear how the student obtained a solution of 38 blocks.

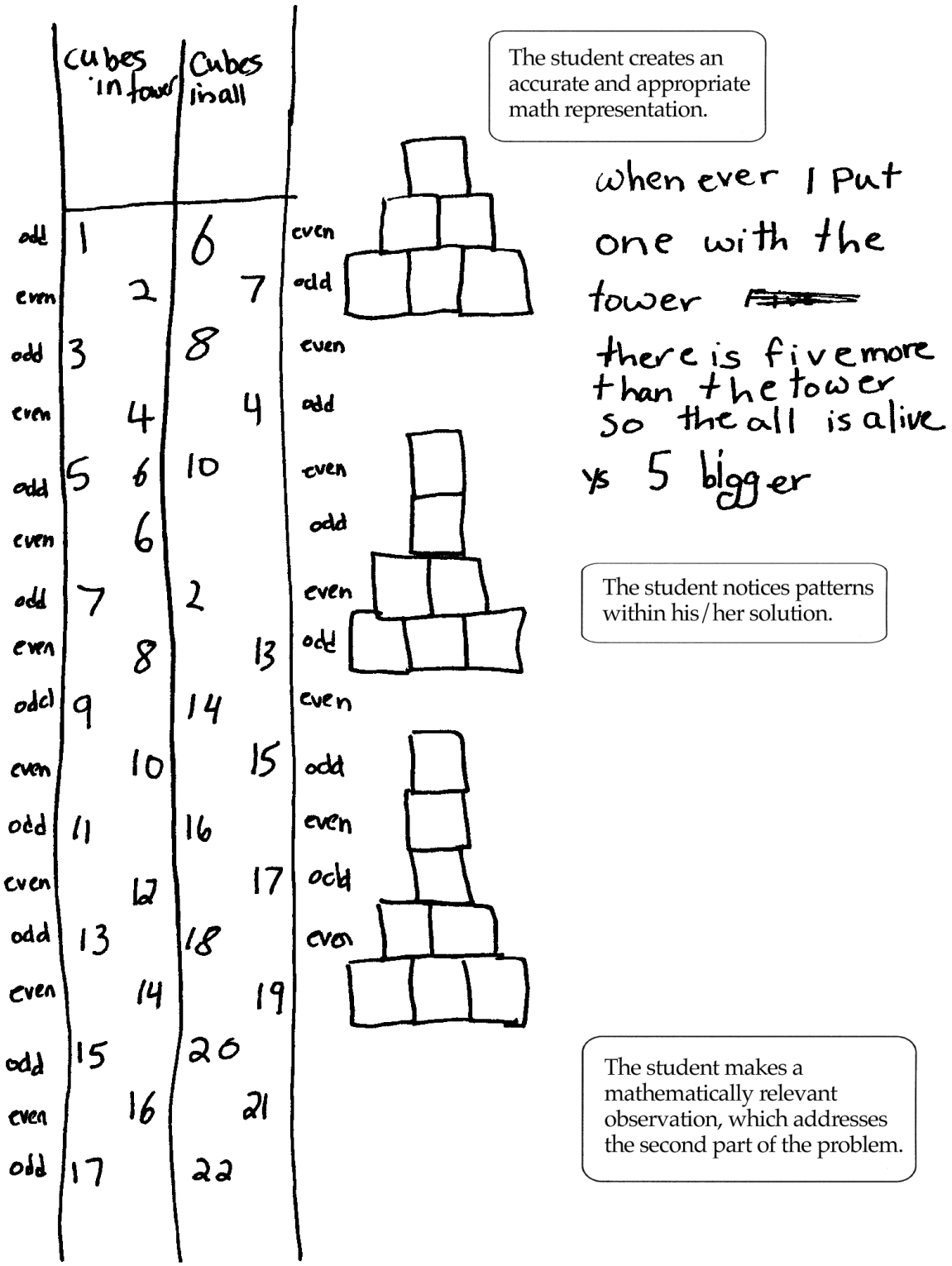
IN 5 towers ther are 38 BLOCKS
I is odd and wow is ever

Apprentice



The Tower Problem

Practitioner



Cubes in tower	Cubes in all	
odd 1	6	even
even 2	7	odd
odd 3	8	even
even 4	4	odd
odd 5	10	even
even 6		odd
odd 7	2	even
even 8	13	odd
odd 9	14	even
even 10	15	odd
odd 11	16	even
even 12	17	odd
odd 13	18	even
even 14	19	
odd 15	20	
even 16	21	
odd 17	22	

The student creates an accurate and appropriate math representation.

The student notices patterns within his/her solution.

The student makes a mathematically relevant observation, which addresses the second part of the problem.

Expert