Checkerboard Investigation

While watching a couple of students play checkers one day, I wondered, "Just how many squares are on that board?" I thought and thought and came up with a number greater than 200! Is that possible?

Your task is to find out exactly how many squares are on that board. Be creative and thoughtful about your mathematics. Show all of your work, explain all of your thinking and identify all the strategies you use to solve this problem.

Grade Levels 3 - 5

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Context

We had been working with exponents. My students have explored square numbers, working with sugar cubes to build squares and to visualize the growing of patterns.

What This Task Accomplishes

This task requires students to use problem-solving approaches to investigate, draw logical conclusions and generalize solutions and strategies. It requires understanding and applying reasoning processes, with special attention to spatial reasoning. It also allows me to assess whether or not students have understood the activities we have been working on in class.

What the Student Will Do

Most students will start with finding the number of squares with an area of one, then an area of four and so on. Some were not able to approach the task as systematically, which resulted in incomplete solutions.

Time Required for Task

The students were given one week to work on this problem individually or in a group.

Interdisciplinary Links

Students had been studying the history of games. Questions we have explored include: What constitutes a successful game? A challenging game? A game that will endure over time? The unit could be further extended in mathematics to the area of probability and what constitutes a "fair game".

Teaching Tips

Working with sugar cubes to build squares and visualize growing patterns is a helpful pre-assessment activity. I have also taught my students the following five steps of an investigation, which was also useful:

1. Collect your data.
2. Organize your thinking (identify strategies).
3. Represent your thinking using graphs, charts, tables, drawings, etc.
4. Explain your thinking (validate your solution).
5. Refine and/or extend your thinking. (Find another solution. Create another problem.)

Suggested Materials

- Graph paper
- Colored pencils
- Sugar cubes
- Checkerboards
- Squares

Possible Solutions

Total = 204 squares:

8 x 8 = 64 1 x 1 squares
7 x 7 = 49 2 x 2 squares
6 x 6 = 36 3 x 3 squares
5 x 5 = 25 4 x 4 squares
4 x 4 = 16 5 x 5 squares
3 x 3 = 9 6 x 6 squares
2 x 2 = 4 7 x 7 squares
1 x 1 = 1 8 x 8 squares

Benchmark Descriptors

Novice
A Novice's response will demonstrate limited understanding of the complexity of his/her task and merely count the squares traditionally noticed. The student uses neither math language nor representation to communicate.

Apprentice
An Apprentice's solution may not be complete. The student may find the 64 small squares and knows that there were other size squares. However, the student's strategy may only be partially useful since it lacks a systematic approach. An Apprentice may show some mathematical reasoning in finding other sized squares. There is some use of representation and some explanation of strategy using some mathematical terminology and notation.

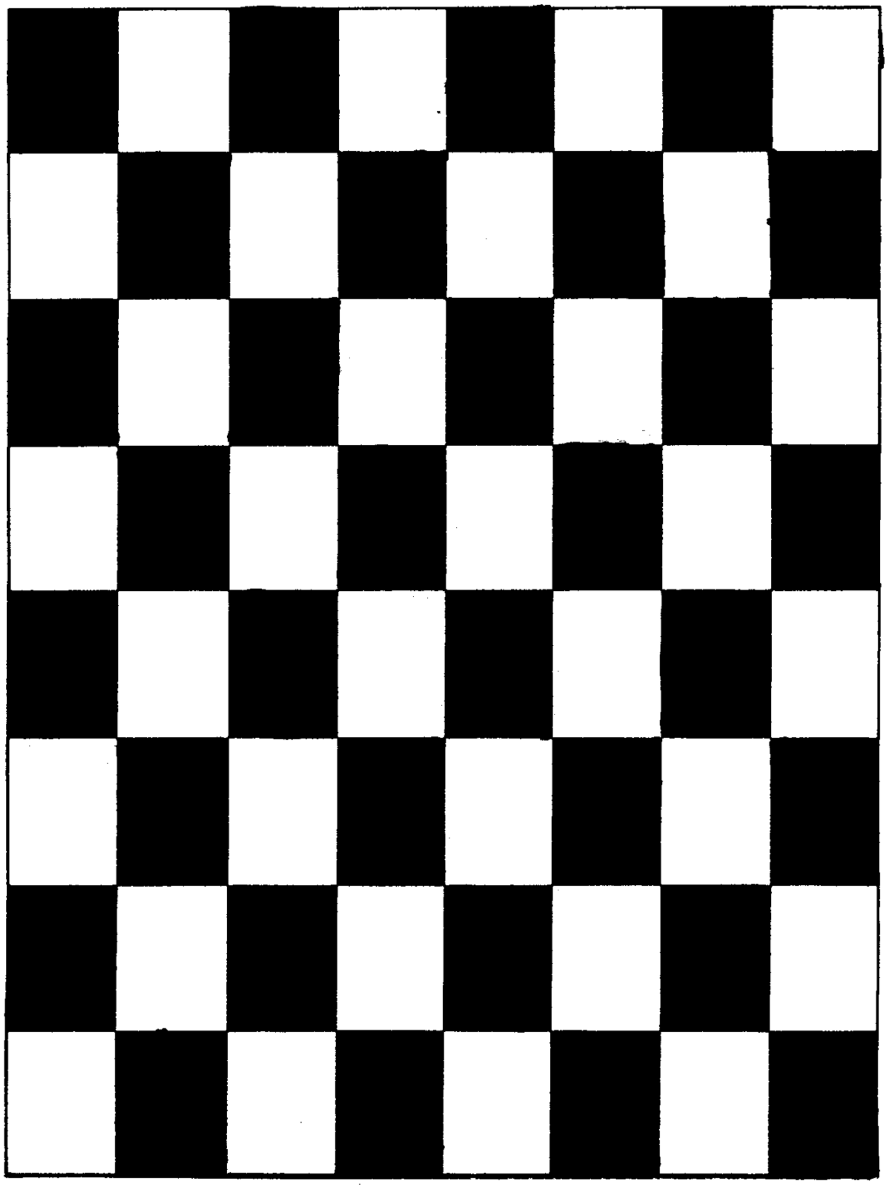
Practitioner
A Practitioner's solution demonstrates understanding of the major concepts needed to find all the squares. The student uses effective reasoning that leads to a solution. There is a clear explanation that uses effective mathematical terminology and notation. The representation is accurate and appropriate for the problem.

Expert
An Expert demonstrates deep understanding of the problem by recognizing the pattern and being able to express this pattern algebraically or exponentially. The representation accurately and appropriately communicates the student's solution.

Novice

add on another 3 squars to make it larger.

You keep moving the perimeter of the square over one unit to create a new square



This student does notice a pattern of always adding an odd number in order after obtaining one solution to find the next.

Student explains part of his/her solution.

to make a bigger squar you would add on 5 more squars. Once you finish you keep going up the odd #table and add whatever number of squars on that you are up to. 3,5,7,9 ect.

Novice

It is possible to have more than 200 squars on a checker board. All you have to do is look and think. What happens is that if you keep adding on squars to the original or make them one size bigger you eventually only have one left. If you start ~~out with~~ 64 squars the number will decrice by 5 every time you make the squar one size bigger. When you add up all the amount of squars from each different size of squars you will get over 200.

going down the odds tables

My only strategy was to stare at the checkerboard and think back to what we did in class. Once I remembered what we did in class the rest was very easy.

It is unclear what the student means by "only one left". The student lacks documentation of his/her work. The student does use some good math language to communicate.

Apprentice

The student uses good math language and clearly explains his/her thinking.

Representations are clear and accurate. The student's work lacks documentation. How does the student get 220 squares?

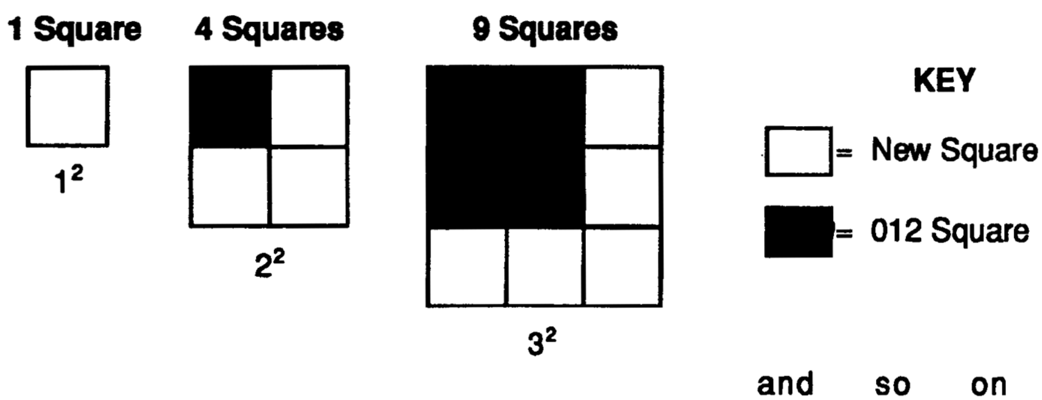
POW #5
THE CHECKERBOARD INVESTIGATION

I think it is possible for a checkerboard to have a number greater than 200 because besides the normal squares that are played with there are other squares made up of more than one square.

EXPLANATION

A checkerboard has 64 squares that are always played, but if you add three squares onto each single square you will get slightly larger square made of four single squares and if you add five more single squares onto the square made four singles it will yet again get one row larger and you continue to add single squares consecutively up the odds table you will see each square become one row larger.

EX



I found out by using multiplication and subtraction it is possible for checkerboard to have more than 200 squares. The approximate answer is there are 220 squares on a checkerboard.

STRATEGIES USED: Logical thinking and solve a simpler problem.

Practitioner

The total number of squares on a checkerboard is 204. How did I figure this out you ask Well, I looked beyond the obvious. If somebody looked at a checkerboard they would probably say there are 64 squares by figurines 8 rows x 8 columns = 64 squares Wrong! As I said before, the answer is 204. And I'll explain...

By looking beyond the obvious I mean I realized that individual squares couldn't be all I was looking for I used creative thinking.

I kept increasing the size of the squares were 1 column by 1 row. That number is 64 for 8 columns 8 rows.

2 columns by 2 rows = 49 squares. I found this by drawing a checkerboard and using my hands.

Student obtains a correct solution. The student uses accurate and appropriate math language.

Practitioner

to cover the squares. I was trying to count.

In this part of my problem solving I used the strategies of drawing a picture and using objects.

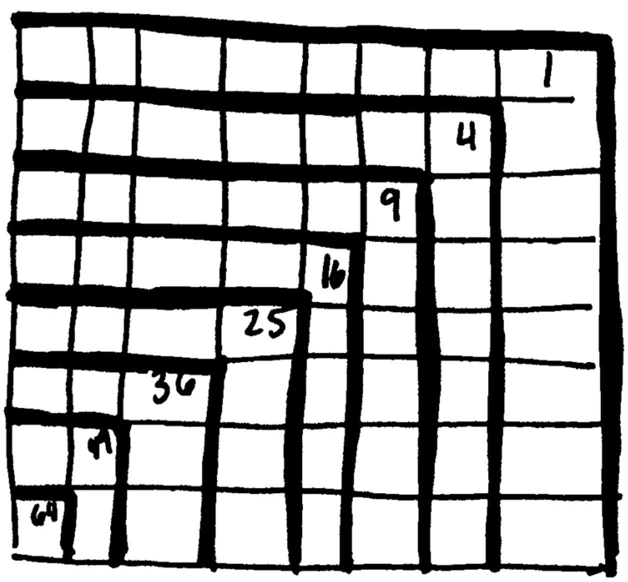
The student makes a mathematically relevant observation.

I then noticed that as I increased the size of the squares by 1 column and 1 row I decreased the number of column and rows by one that I was able to use. For example, 1x1 squares used 8 columns and 8 rows, 2x2 used 7 columns and 7 rows and so on. In this part of the problem solving I found a pattern.

Then I used logical thinking and guess and check strategically guessing that a 3x3 square would use 6 rows and 6 columns to create 36 squares (6x6).

Practitioner

You can see how the total number of squares were computed using this method on the chart that follows.



64
49
36
25
16
9
4
+ 1

204 total squares

The student creates an accurate and appropriate math representation to communicate his/her solution.

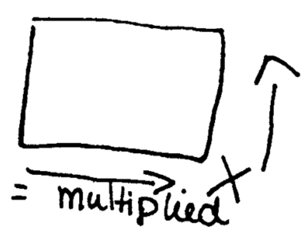
Expert

The student demonstrates a command of math language.

The student uses a systematic strategy to obtain a solution.

The first thing I did in starting this POW was to find out how many single squares there were. I did that by finding the area. I multiplied the number of squares going horizontal on one side I got 8. I multiplied that by the number of squares going vertical on one side. I got 8 I multiplied 8x8. I got 64 so there are 64 single squares on a checkerboard. I made a chart.

Squares	#
single	64



I then found out the number of 2 by 2 squares on the board. I found out how many squares there were going vertical on on side I got 7. I found out how many squares there were going horizontal on one side.

Expert

I multiplied 7x7. I got 49.
There are 49 2 by 2 squares
in a checkerboard. I made
another chart.

Squares	#
Single	64
2 by 2	49

The student verifies the correctness of his/her strategy. The student finds an efficient strategy and uses it to solve the problem.

I started to see a
pattern. It looked like
decreasing squares. It
looked like the single squares
was 8^2 the 2 by 2 squares
were 7^2 and so on.

I tried once more just
to be sure. I counted how
many 3 by 3 squares there
were. I got 6 going horizontal
on one side and 6 going
vertical on one side. I got
$6 \times 6 = 36$ or 6^2. There are 36
3 by 3 squares on the
board.

Expert

I made another chart.

Square	#
Single	$64=8^2$
2 by 2	$49=7^2$
3 by 3	$36=6^2$

I just filled in the rest.

Square	#
Single	$64=8^2$
2 by 2	$49=7^2$
3 by 3	$36=6^2$
4 by 4	$25=5^2$
5 by 5	$16\ 4^2$
6 by 6	$9\ 3^2$
7 by 7	$4\ 2^2$
8 by 8	$1=1^2$

The student clearly explains his/her strategy.

I added all of the #s up.

Expert

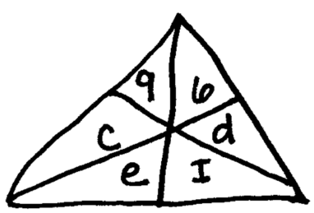
4
64
49
36
25
16
9
4
+ 1
204

I got that there are 204 squares on a checkerboard using decreasing squares.

The student extends the problem by creating a similar problem and solving it.

Extention-

For the extention I basically used the same method but I couldn't find a pattern.

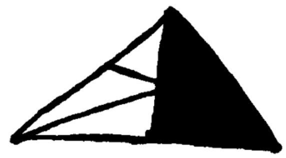


I first counted how many singles triangles there were. I got 6.

I then counted the doubles. I counted 3

Expert

I then did the tripples



1

2.

3.

4.

5.

6.

There are 6 tripples

I then added all of
these up the
one big whole triangle.

Expert

6 singles
3 doubles
6 tripples
+ 1 big one
16

There are 6  triangles

The strategies I used were.

- Construct a chart or table
- Find a pattern