Staircase Dilemma

I am planning to build a staircase. I am not sure how many steps high I want my stairs. I do know that a 1-step stair takes 1 block to build. A 2-step stair takes 3 blocks and a 3-step stair takes 6 blocks.

How many blocks will a 10-step stair use?

Find a pattern to your stairs and if you can, generalize your pattern so I would be able to find the number of blocks in any step of stairs.

Be sure to explain your reasoning.

Grade Levels 6 - 8

Staircase Dilemma

I am planning to build a staircase. I am not sure how many steps high I want my stairs. I do know that a 1-step stair takes 1 block to build. A 2-step stair takes 3 blocks and a 3-step stair takes 6 blocks.

How many blocks will a 10-step stair use?

Find a pattern to your stairs and if you can, generalize your pattern so I would be able to find the number of blocks in any step of stairs.

Be sure to explain your reasoning.

Context

I am trying to get my students to think about finding generalizations when they solve a problem. I have been giving them problems like "What's My Rule?" where they have to see a pattern in a set of numbers and try to express that pattern in an equation using letters for the variables. In the beginning of the year I gave my students the "Handshake Problem" and was very pleased with the results. They observed many patterns, but few could come to a generalization. The "Staircase Dilemma" is really a version of the "Handshake Problem". I wanted to see how many students could connect the two problems and to see if more students were able to come to a generalization. I was very pleased with the results - and very pleased with the effort students made in trying to turn patterns into generalizations. Many could not believe they were using the same strategy as the strategy they used in the "Handshake Problem". They do not know it yet, but later on this year they will see this problem again presented geometrically.

What This Task Accomplishes

Because this task is similar to the "Handshake Problem" I was able to compare a student's solution to the "Staircase Dilemma" to their "Handshake Problem" solution and look for growth in their problem-solving, reasoning and communication skills.

What the Student Will Do

The student will begin by drawing staircases. Some will make a chart to keep track of the stairs and blocks. Some used the chart to help them find a pattern and a generalization. Hopefully students will recognize this problem as being a different form of the "Handshake Problem".

Time Required for Task

60 minutes

Interdisciplinary Links

This task can be used with art activities or in a discussion of architecture.

Teaching Tips

Working with students to help them understand the importance of generalization in mathematics and ways of recognizing patterns will help prepare them for this and other problems. If you have not yet used the "Handshake Problem" included on this CD you might try that problem first.

Suggested Materials

Centimeter graph paper

Possible Solutions

A 10-step staircase will take 55 blocks.

Students who use a chart of simpler cases should be encouraged to look for patterns.

Number of Stairs	Number of Blocks
1	1
2	3
3	6
4	10
5	15
6	21
.	.
.	.
10	55

A generalization for this problem is:

(number of steps) x (number of steps +1))/2

They may also come out with an infinite series where N is number of steps in staircase and B is the number of blocks that looks like:

B = N + (N -1) + (N - 2) ...... +1)

Novice
There is no explanation of a solution presented. There is no use of mathematical terminology and the diagram is not accurate. The student will not be able to see a pattern nor come to a

generalization with this solution.

Apprentice
The solution is not complete - the student did not mention the pattern that was being described as the strategy. The strategy did, however, lead the student to the number of blocks in the 10-step staircase. There is some use of appropriate mathematical representation and some use of mathematical terminology.

Practitioner
This solution shows the student has a broad understanding of the problem and the major concepts necessary for its solution and uses effective mathematical reasoning. The student uses a strategy that leads to finding a pattern and talks about a generalization. The generalization is not expressed as an equation, but a connection to the "Handshake Problem" is made. The student has a clear explanation, uses appropriate mathematical representation, and there is effective use of mathematical terminology and notation.

Expert
This solution shows a deep understanding of the problem including the ability to identify a sophisticated mathematical generalization. The student uses an efficient and sophisticated strategy leading directly to the generalization. The student connects multiplication to rectangles and then sees the staircase as half the area of that rectangle. There is a clear and effective explanation - this is a good example of mathematical representation to actively communicate ideas related to the solution.