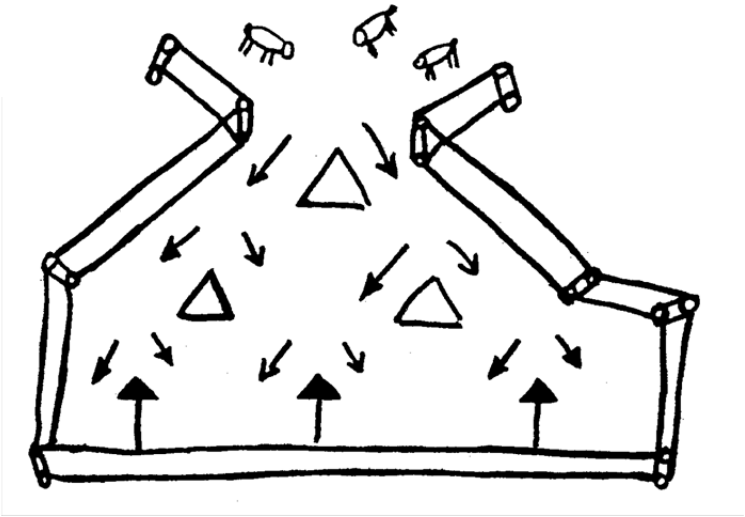
Sheep Herder's Dilemma

Mr. Frye has 750 sheep that he needs to corral for shearing. He had an ingenious way of dividing up his sheep. He built a big corral with triangular obstacles that divided the sheep as they walked through the corral. Each time a group of sheep met an obstacle, 1/2 would go to the right and 1/2 would go to the left. Find out about how many would end up in each pen at the end.

Be sure to explain your reasoning clearly. Connect as much mathematics and use as much math language as you can. Make any generalizations that you can.



Grade Levels 6 - 8

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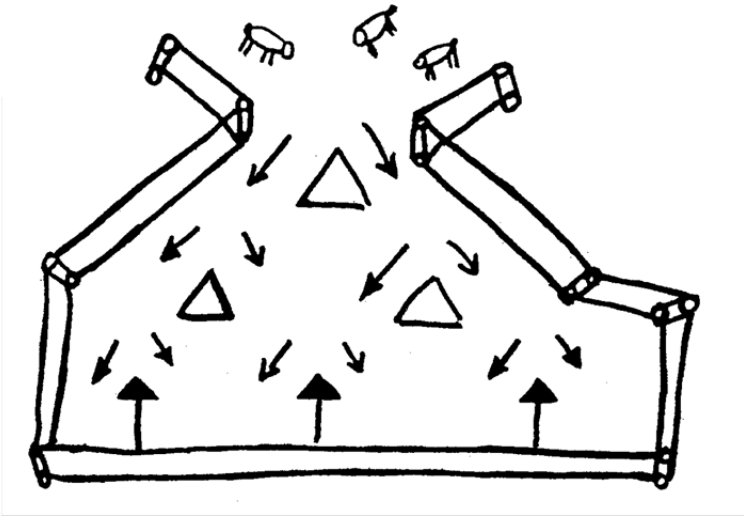


Context

I had been to a science museum and saw an activity where they let a large number of ping-pong balls cascade through a set up with barriers similar to the diagram in the problem. I thought it would be an interesting problem to pose for my sixth graders and see what they could do with it. I figured it was simple enough that just about everyone could engage in the problem, but I also wanted to see if some students could take it further and come up with some generalizations. It was not until after solving the problem and talking the solutions over with my students that I saw the connection to Pascal's Triangle. My students have not investigated Pascal's Triangle and could not make the connection, but some students in later grades may see the pattern.

What This Task Accomplishes

Although not terribly difficult mathematically, this problem allows students to reach for a

generalization. Students also need to decide how to divide an odd number of sheep (some amazingly said 187 1/2 sheep go each way). I also wanted them to see the bell curve affect when this pattern is followed. This was contrary to their first thought that each pen would have the same number of sheep at the end.

What the Student Will Do

Most students began by dividing the 750 sheep by two and continued to divide the sheep by two. Some students could not correctly interpret what happened to the sheep in the middle as they divide, but then join other divided sheep. Some could not drop the fact that they thought the pens should have the same number of sheep and made their math work that way. Some students noticed a pattern and worked for a generalization.

Time Required for Task

45 minutes

Interdisciplinary Links

This is really a mathematical problem, although it could be related to traffic patterns or in some way to farming.

Teaching Tips

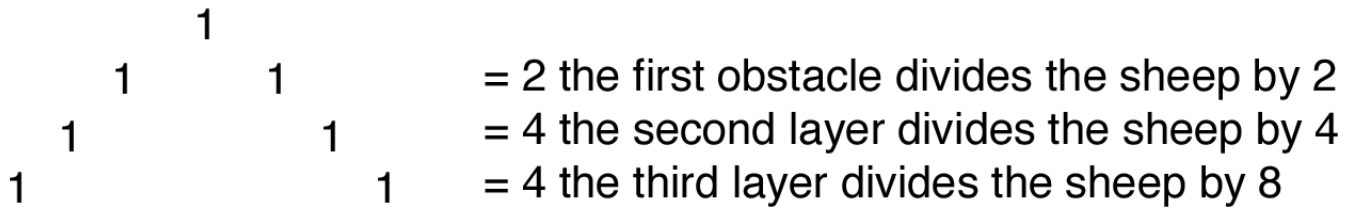
If your students have studied Pascal's Triangle, it would be fun to see if they recognized the pattern in this problem. You may also want to have another layer of obstacles. I thought maybe if I had a row with three obstacles, that they may have seen and commented on the fact that exponents could be used to express the fractional part of the sheep following the outside path (first 1/2 of the sheep go to the outside then 1/4 of the sheep go to the outside, then 1/8 of the sheep and finally - if there was another layer - 1/16 of the sheep go into the outside pens).

Suggested Materials

Graph paper (may be needed)

Possible Solutions

The two outside pens will have 93 or 94 (depending on how they divided the odd number of sheep) and the inside pens will have 281 or 282 sheep - for the same reason. You can also relate it to Pascal's Triangle (see the graphic below).



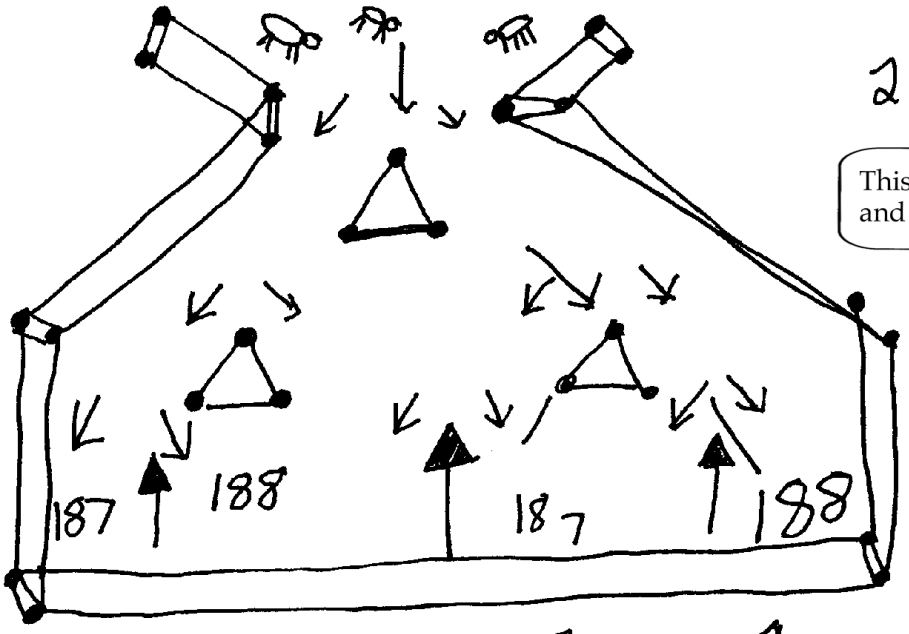
Novice
The student applied inappropriate procedures to solve the problem. S/he took the three obstacles and multiplied by two ("because each time they break in half") and then divided 750 by six. This strategy will not help to solve the problem. There is no evidence of mathematical reasoning.

Apprentice
This student used a strategy that is partially useful, leading some way toward the solution, but not to a full solution. His/her explanation also shows that there are parts of the problem s/he does not understand completely. However, his/her rejections of strategies tried to show some evidence of mathematical reasoning. S/he divides the 750 sheep in half and then successfully divides the second row of sheep. S/he fails, however, to mix and then divide the third row of sheep as they go into their pens. S/he makes an attempt at using the diagram, but makes mistakes on recording the number of sheep on each side of the obstacles.

Practitioner
The solution shows that the student has a broad understanding of the problem and the major concepts necessary to solve the problem. S/he correctly estimates that there will be different amounts of sheep in the center pens than the outside pens and divides the number of sheep in a reasonable way. The strategy leads to a solution and s/he uses effective mathematical reasoning. S/he checks the work to be sure the pens hold 750 sheep. There is a clear explanation and the diagram shows more clearly the distribution of sheep. The mathematical terminology and notation is appropriate and correct.

Expert
This student's solution shows a deep understanding of the problem including the ability to generalize the solution. S/he uses a strategy that leads directly to the solution and employs refined reasoning in coming up with a relationship between the number of sheep in the pens and the original number of sheep. There is a clear and effective explanation detailing how the problem was solved. The circle graph helps communicate the relationship between all the sheep and the sheep at any level.

Novice



1.87
2 | 3.75

This strategy lacks reasoning and will not lead to a solution.

Exact
188 ≠ 187

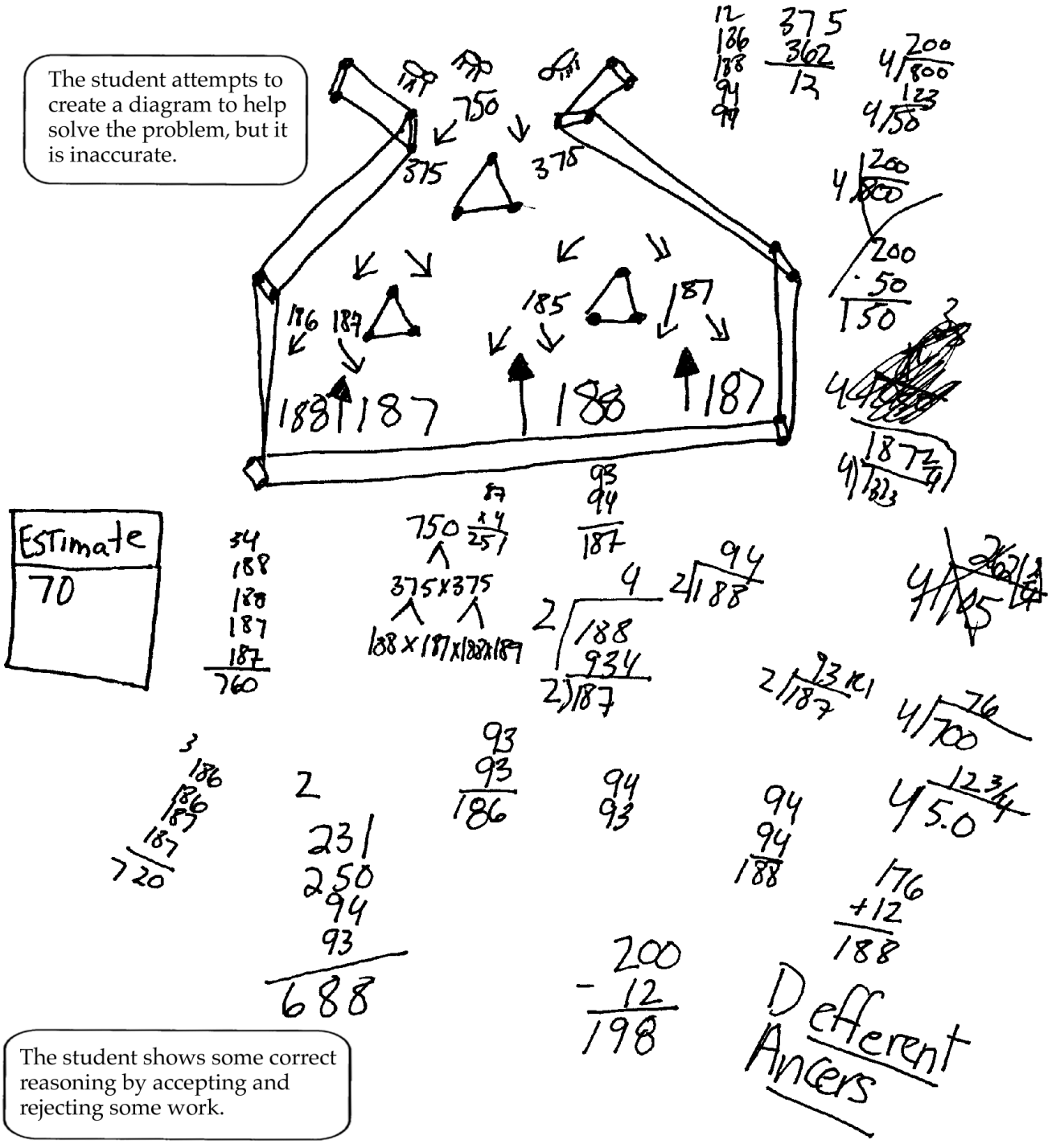
Estamate

6 | 125
250
3

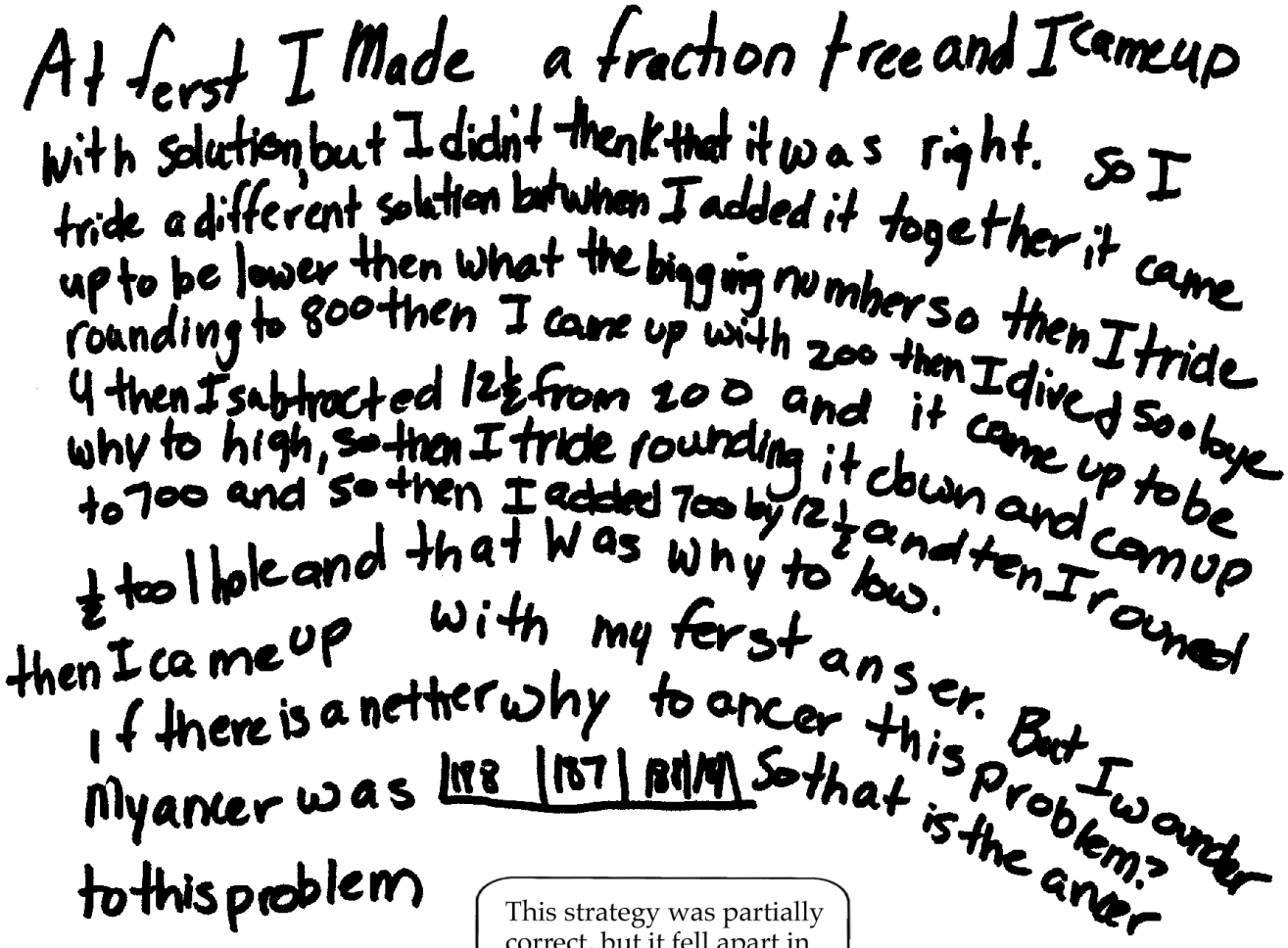
The student explains his/her incorrect reasoning.

My estamate was veary differnt from my answ. I got my estamate by deviding 750 by 6 because there is 3 obstides and each time they break in half so I would devide it by 6.

Apprentice

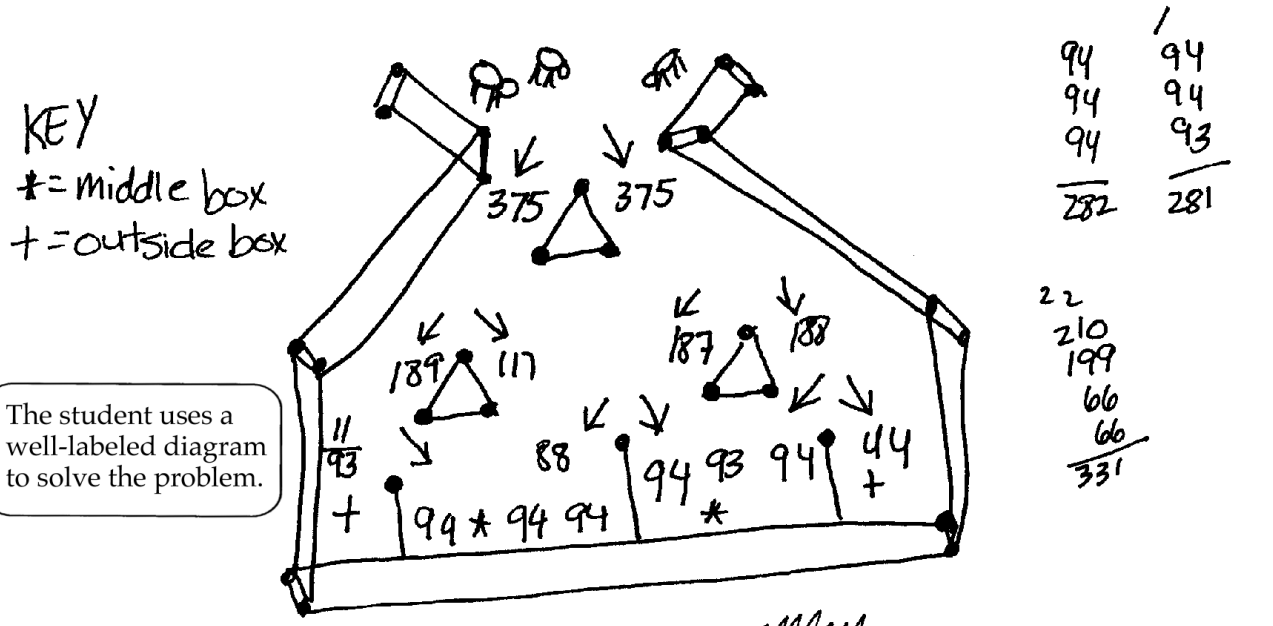


Apprentice



This strategy was partially correct, but it fell apart in the third row.

Practitioner



The student uses a well-labeled diagram to solve the problem.

Estamate there will be twice as many in the two middle boxes then in the two outside boxes. The middle boxes have three times more then the other boxes.

Diagram on back →

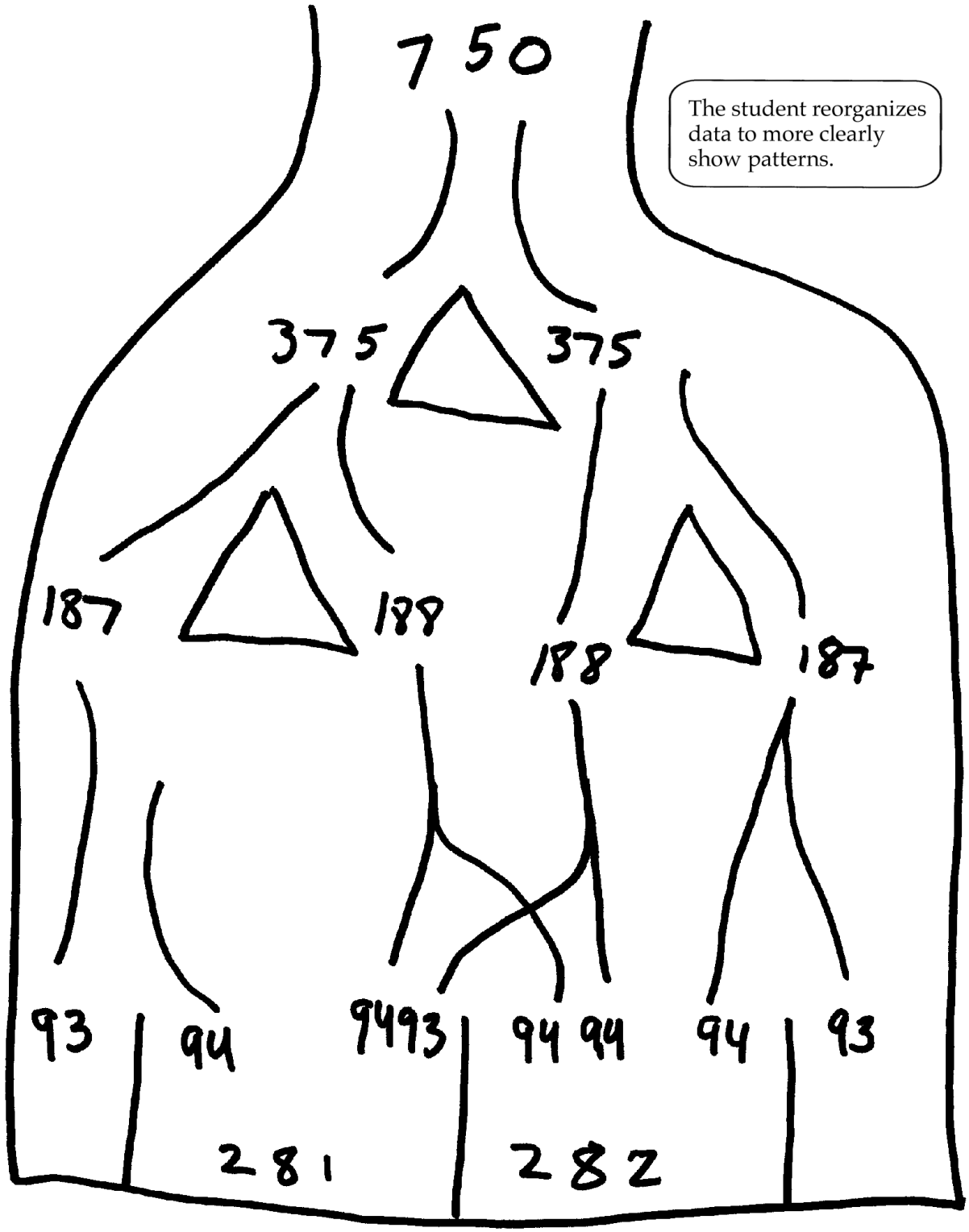
This student clearly explains his/her approach and reasoning.

There is in the middle pens there are 282 or 281. In the other pens there are 94 and 93. The reason there are different numbers for the same pens is two of the numbers are even

This shows a good problem-solving strategy.

I found the answer by taking it so dividing it in half and got 375 then I divided 375 in half and got 187 and 188 then I divided those and got 93 and 94. Then I added them all up and got 780.

Practitioner



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Expert