Handshake Problem

If everyone in the class shook hands with everyone else, how many handshakes would there be?

As you solve this problem, be sure you explain clearly your reasoning. Use some kind of representation to help explain your reasoning. Make comments about any patterns you see.

Grade Levels 6 - 8

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Context

This task can be given to students with some sense of number and patterns. Students with knowledge of the area of rectangles and triangles may make some strong connections that allow them to come to a generalization of the solution.

What This Task Accomplishes

This problem makes a powerful statement to students that the use of diagrams and charts can help make connections to patterns and generalizations. It also shows students that acting out a problem often helps formulate a strategy that can solve the problem.

What the Student Will Do

As students solve this problem, they may very well act out the problem. Others will break the problem down to simpler cases and look for a pattern. Students will often begin by making some kind of drawing, chart or diagram to help them sort out the problem. Often times there is a discussion of what constitutes a handshake. Students need to come to a resolution as to whether when two people shake hands it counts for one or two handshakes.

Time Required for Task

60 minutes

Interdisciplinary Links

This task can be used to demonstrate a number of different lessons. In science or health it can show how disease spreads across a population. It could be useful to show how AIDS has become so widespread among some populations.

It might also be used in social studies in a discussion of how people greet one another in different societies or simply to demonstrate the "social niceties".

Teaching Tips

If the question of what a handshake is comes up, allow students to give their opinions and try to convince others of their point of view. Students can be given a great deal of latitude in solving this problem - there are a number of strategies that are appropriate, from acting out the problem to using more refined approaches.

Suggested Materials

- Graph
- Lined and blank paper
- Pencil
- *Compass

*Some students see people standing in a circle and each person is a dot on the circle and gets connected to every other dot.

Possible Solutions

Students who use a chart of simpler cases should be encouraged to look for patterns.

Number of People	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
6	15
.	..
.	..
.	..

Students who use the rectangular representation (see Expert Benchmark) should be encouraged to connect their understanding of area to simpler cases to come up with a generalization.

(number of people) x (number of people - 1)) / 2 = handshakes

Benchmark Descriptors

Novice

This student used an inappropriate approach. Finding the number of handshakes at each group of desks and adding the results together will not lead to a correct solution. The representation of

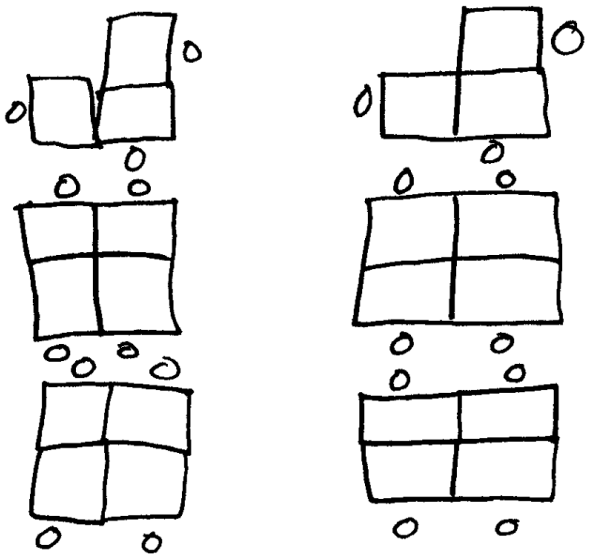
the desks and students at the desks does not help solve the problem.

Apprentice
This student had a strategy that was partially useful. They left out the last handshake. There is some evidence of mathematical reasoning. The solution lacks a complete explanation.

Practitioner
The solution and comment on finding a pattern show that the students have a broad understanding of the problem. They use a strategy that leads to a solution using effective mathematical reasoning. There is a clear explanation and appropriate use of a chart.

Expert
The solution and generalization show a deep understanding of the problem. Using a formula to solve the problem is a sophisticated and efficient strategy that leads directly to the solution. The solution shows refined and complex reasoning. There is a clear and effective explanation of the solution. The mathematical representation is actively used as a means of communicating ideas related to the solution of the problem.

Novice



All diagrams are rudimentary and did not assist in finding a solution.

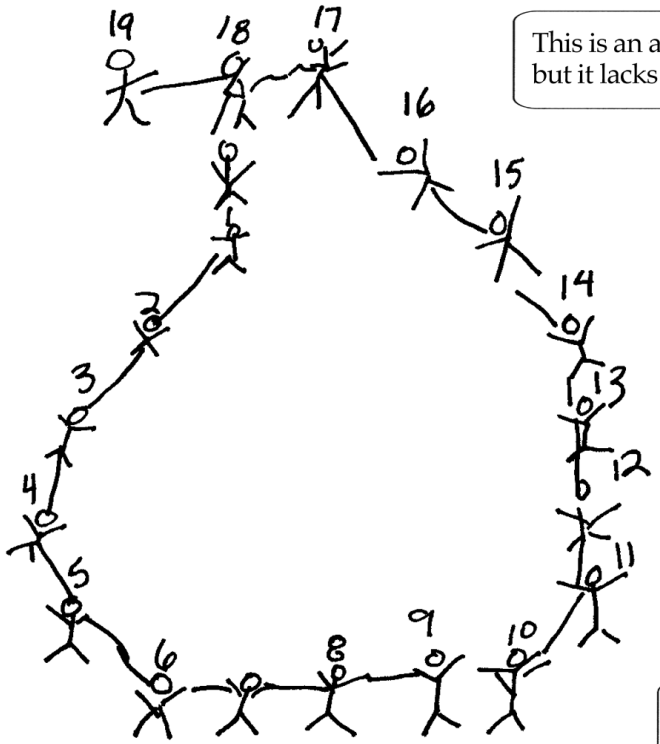
The student uses little or no math language.

tables with 4 people have 6 handshakes
tables with 3 people have 3 handshaks

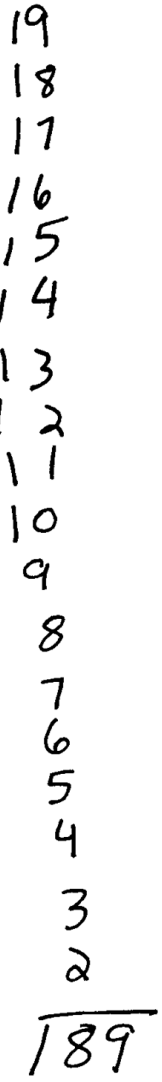
3
3
4
4
4
+ 4
22 handshaks

The student does not have an approach that would work.

Apprentice



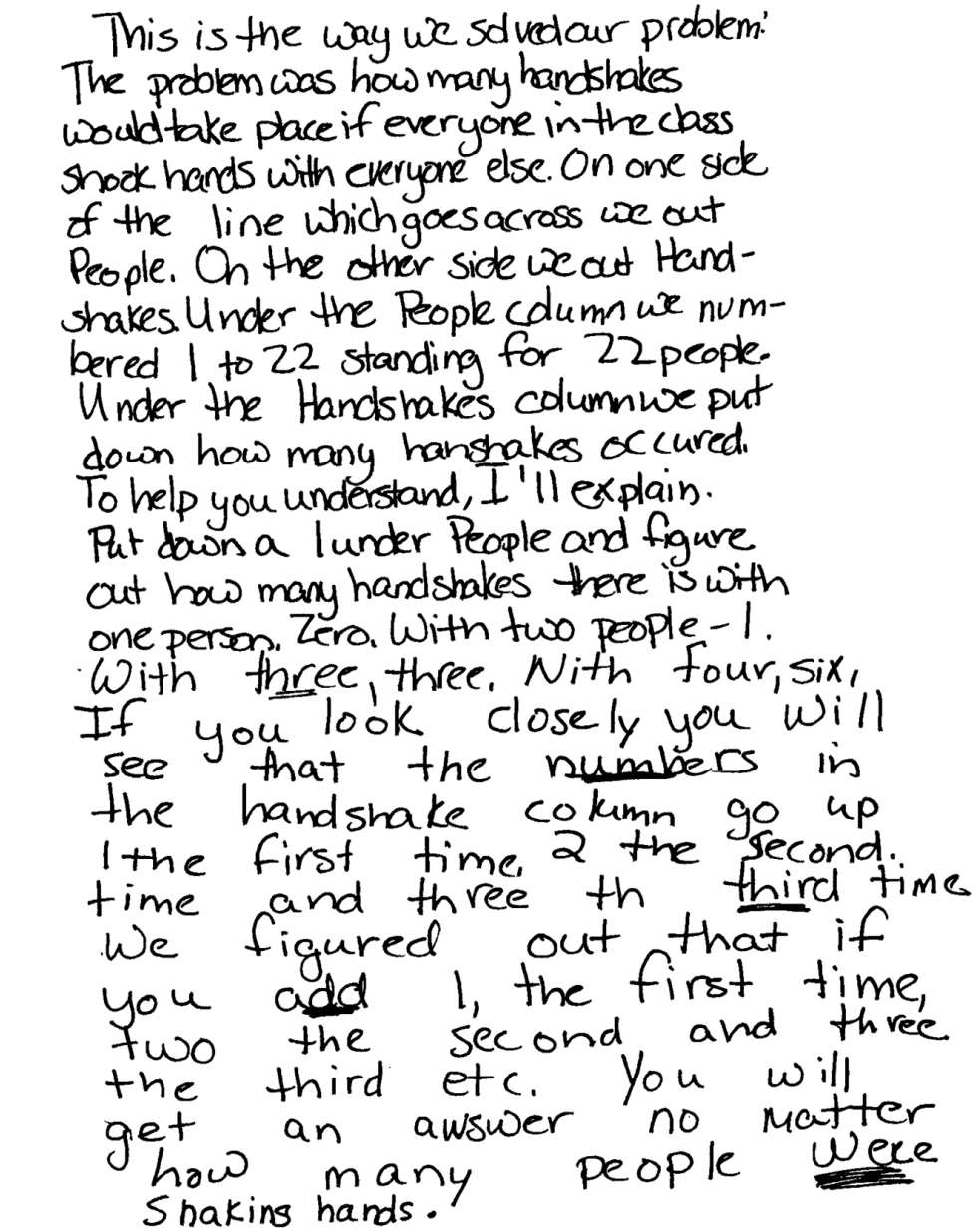
This is an attempt at a diagram, but it lacks labels.



The student uses no math language to communicate.

The student has an approach that would work, but one handshake is left out.

Practitioner



This is the way we solved our problem. The problem was how many handshakes would take place if everyone in the class shook hands with everyone else. On one side of the line which goes across we out People. On the other side we out Handshakes. Under the People column we numbered 1 to 22 standing for 22 people. Under the Handshakes column we put down how many handshakes occured. To help you understand, I'll explain. Put down a 1 under People and figure out how many handshakes there is with one person. Zero. With two people - 1. With three, three. With four, six, If you look closely you will see that the numbers in the handshake column go up 1 the first time, 2 the second. time and three th third time We figured out that if you add 1, the first time, two the second and three the third etc. You will get an awswer no matter how many people Were Shaking hands.

The identification of the pattern leads to the discovery of a rule.

The student uses some math language to communicate.

The student identifies a pattern in the solution.

Practitioner

People	Handshakes
1	0 → +1
2	1 → +2
3	3 → +3
4	6 → +4
5	10 → +5
6	15 → +6
7	21 → +7
8	28 → +8
9	36 → +9
10	45 → 10
11	55 → 11
12	66 → 12
13	78 → 13
14	91 → 14
15	105 → 15
16	120 → 16
17	136 → 17
18	153 → 15
19	171 → 19
20	190 → 20
21	210 → 21
22	231 → 0

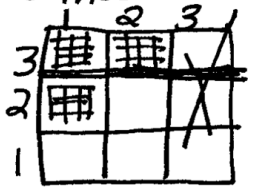
The student attempts to create a representation, which lacks labels.

Expert

The students use a sophisticated strategy.

The students use a diagram to solve the problem.

We looked at our graph and saw that half of the rectangle was shaded. We tried a Few smaller numbers like 3



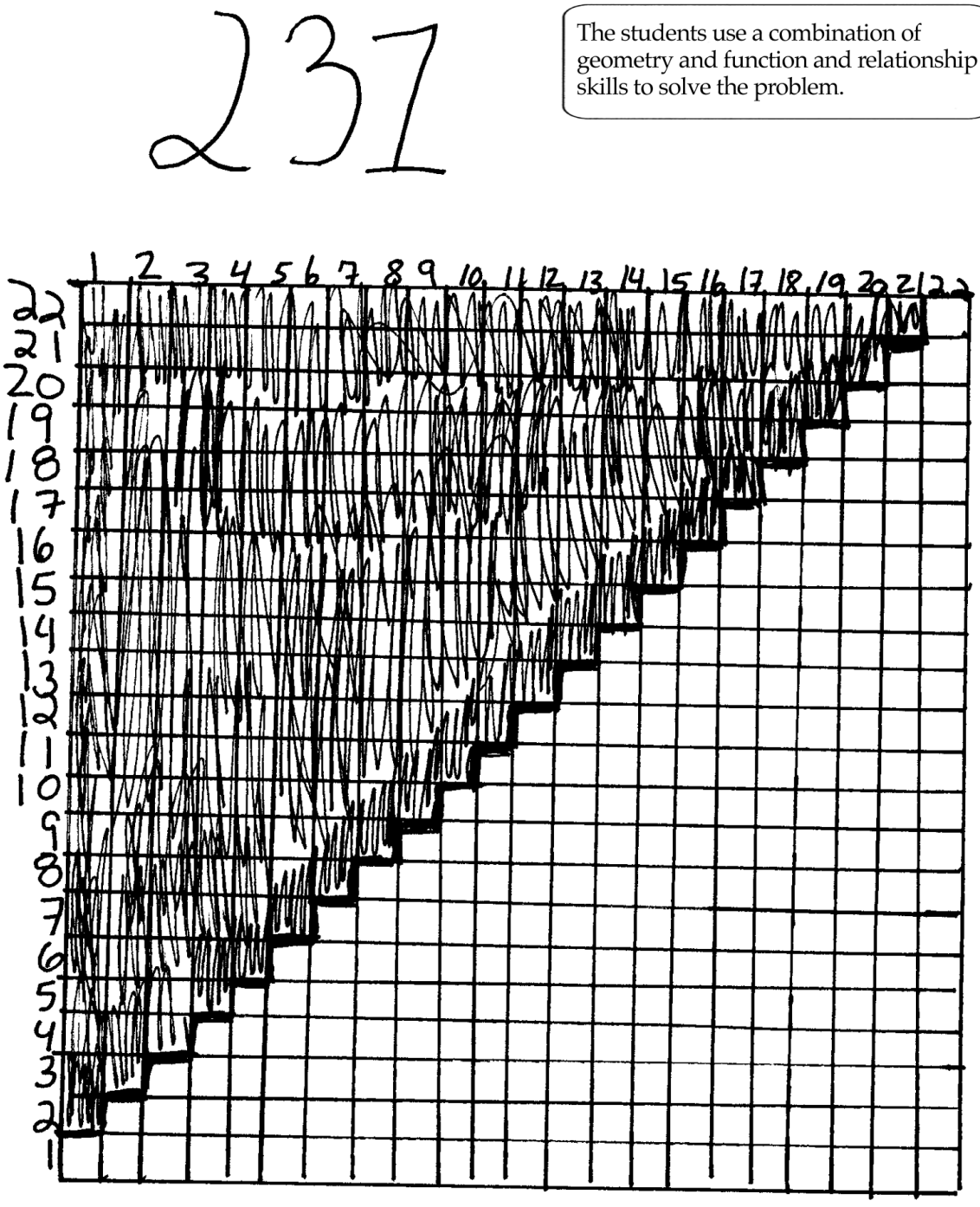
Take off the end and you have a 2by3 rectangle and 3 squares are shaded — that's half. We tried a few more and it worked we think you can multiply the number of kids times the one les and take half it that number.

Like : Kid X(Kid−1)÷2

The students create a general rule for solving the problem.

The students test and accept their hypothesis.

Expert



The students use a combination of geometry and function and relationship skills to solve the problem.