Double Dilemma

Mrs. Callens and I were talking about board games the other day. She said she and her daughter and son were playing Monopoly® over the weekend. In Monopoly®, you get an extra turn when you throw a double. She said she was really lucky in those types of games because she rolled doubles about 1/3 of the time.

Show Mrs. Callens how much you know about probability and either have evidence that proves she was lucky that day or that it was not luck - that anyone would expect to throw doubles 1/3 of the time.

Grade Levels 6 - 8

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Context

We were studying probability in my sixth grade class. With partners in class we experimented with pulling a colored chip from a bag that had three blue chips, two red chips and one yellow chip. We talked about the experimental and theoretical probabilities and how to record each probability. We also predicted and experimented with tossing two coins. We discussed why HT came up twice as much as TT or HH.

What This Task Accomplishes

I wanted a problem that required students to apply their knowledge of probability. I also wanted to assess their understanding that like the coin toss, sometimes you need to distinguish between the two coins. In the "Double Dilemma" they need to find the total possibilities and remember that rolling a one and a two is different from rolling a two and a one. I also wanted to see how comfortable they were with both experimental and theoretical probabilities. I was hoping that they would include both experimental and theoretical probabilities in their solutions.

What the Student Will Do

Some students began listing all the possible throws to find the theoretical probability. Others began with rolling two dice and finding the experimental probability. The question of whether to consider one or two ways of throwing a three came up. When they asked me if there was a difference, I simply told them that was a decision that had to be made, but that I was not going to make it for them.

Time Required for Task

Two, 45-minute periods

Interdisciplinary Links

This task can be used with a science unit on weather. For example, how does a meteorologist decide the probability of rain?

Teaching Tips

I happen to have colored die - this fact helped some students remember that a three could be thrown with a one and two as well as a two and one. As I gave out the die - some students specifically asked for two different colors. Graph paper should be available for graphing, as well as a compass and protractor, in case some students want to make a circle graph.

Suggested Materials

- Colored die
- Graph paper
- Compass
- Protractor

Possible Solutions

The theoretical probability of throwing doubles is six out of 36 combinations so:
P (doubles) = 6/36 = 1/6 = .166... = 16 2/3%. Mrs. Callens was lucky that day since she rolled doubles 1/3 = .333... = 33.33...% or 33 1/3% - about double the expected rolls.

Benchmark Descriptors

Novice
This student experimented with two die and kept track of the doubles that were thrown. However, there is no evidence that this student understood the problem. They do not appear to connect their results to Mrs. Callen's results. There is no probability language; the chart to keep track of their throws is minimal. There is no explanation of their strategy or why this experiment would help solve the problem.

Apprentice
This student's response is a typical Apprentice response. They did not understand that throwing a three could be a one and two or a two and one. They only found 21 possible combinations. This student even found an experimental probability very different than his/her theoretical probability and did not realize that something must have gone wrong. Their strategy - finding the theoretical and experimental probabilities - is good, but their lack of understanding of probability, and their lack of understanding of their results, showed poor reasoning skills. There is no mathematical representation, although their mathematical language is good.

Practitioner
This student has a good understanding of theoretical and experimental probabilities. Their strategy leads to a solution. They use effective reasoning. They compare their experimental probability to Mrs. Callens's results. Then they figure the theoretical probability correctly. Their

last statement compares these results and concludes that Mrs. Callens rolled twice as many doubles as expected. They use accurate probability language throughout although there is no mathematical representation. The explanation is clear.

Expert

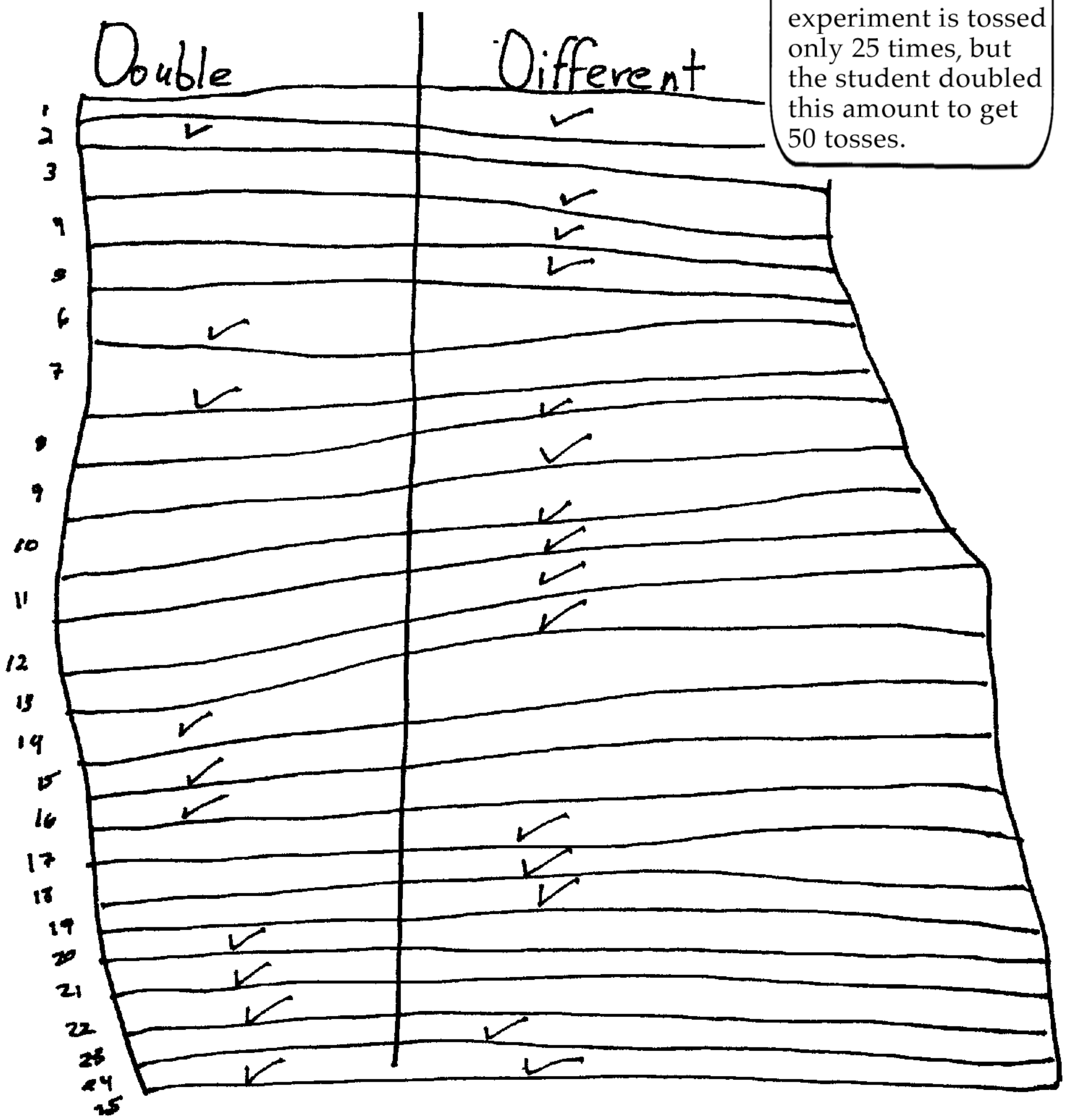
This student has taken the problem and found a generalization for harder cases. They are finding the probability for throwing doubles, triples, quadruples, etc. This student has a deep understanding of probability. By finding a generalization, they are finding an efficient and sophisticated strategy that leads directly to the solution. This student is using refined and complex reasoning. There is a clear and effective explanation detailing how the problem was solved. There also is precise and sophisticated mathematical representation and notation (use of exponents and circle graphs). This response is certainly a high Expert. I was thinking a student that pulled together theoretical probability, experimental probability, good language and made a circle graph (just a little harder than a bar graph) comparing the theoretical, experimental and Mrs. Callen's results would probably have the making of an Expert.

Novice

Do	Di
20	30
50	50

No evidence that the student understood they needed to compare results with Mrs. Callen's result.

The student seems to think that the same results will happen if they tossed a coin 25 more times. The experiment is tossed only 25 times, but the student doubled this amount to get 50 tosses.



Apprentice

Double Dilemma

First I wrote down all the possibilities of rolling 2 die Then I found all the doubles and they are 1-1, 2-2, 3-3, 4-4, 5-5, 6-6 which totals 6 altoghter or 6 out of 21 or 28% look bellow

The next thing I did was I roled the die 100 times and found the we had 13 doubles out of 100 roles or 13% out of 100

So I concluded that She was alittle luckey if you at theory. But practically there was a great variation.

Poor reasoning. This student notices the difference between theory and experimental probability, but fails to think there is a problem.

1-1	2-2	3-3	4-4	5-5	6-6
1-2	2-3	3-4	4-5	5-6	
1-3	2-4	3-5	4-6		
1-4	2-5	3-6			
1-5	2-6				
1-6					

The student does not consider 1-2 and 2-1 as different combinations.

each colom starts with adouble

The P(of doubles) is $\frac{1}{6}$

Apprentice

1-1 1-2 1-3 1-4 1-5 1-6
~~2-1~~ 2-2 2-3 2-4 2-5 2-6
~~3-1~~ ~~3-2~~ 3-3 3-4 3-5 3-6
~~4-1~~ 4-2 4-3 4-4 4-5 4-6
~~5-1~~ 5-5 5-6
6-6

21 posible throws

6 ~~dobb~~ doubles

$\frac{6}{21}$ is not a $\frac{1}{3}$ of 21

it is a $\frac{1}{3}$ of 18

$21\overline{)5100}$ = 0.28
42
180
168
112

.25 = 1/4

$\frac{6}{6}$

6 is a bit more than $\frac{1}{4}$ of 21
but not

Good strategy to find experimental and theoretical probabilities.

$\frac{1}{3} = 33\frac{1}{3}\%$

42 × 4 = 168

→ Theoritically 6 of of 21 gives you
28% not $33\frac{1}{3}\%$ but dose

→ practly we got 13 out of 100

Apprentice

40

6 3
5 5
4 5
2, 3
3. 1
4 2
4·1
4 4 -
3 1
4 2
6 3
1 1
6 4
5 2
6 4
55
5 4
3 1
4 4
2 1
4 2
3 . 2

66
4 3
5 -1
5 2
5 6
6 6
5 3
4 4
6 4
1 1
5 3
6 3
3 2
6 5
2 1
2 1
6 2
6 4
3 4
3 3
3 1

Practitioner

1 | 2,4
2 | 1,2
3 | 1,3
x 4 | 4,4
5 | 3, 4
6 | 6,3
7 | 1,5
8 | 8,4
9 | 2,6
10 | 6,2
11 | 4,2
12 | 5,1
13 | 6,1
14 | 5,2
15 | 6,1
16 | 2,1
17 | 6,4
t 18 | 1,1
19 | 6,4
x 20 | 4,4
21 | 6,5
22 | 6,5
23 | 4,3
24 | 3,5
25 | 3.5

26 3,1
27 1,6
28 4,1
29 2,1
x 30 6,6
31 1,3
32 6,4
33 4,1
34 1,6
35 5,1
36 1,4
36 4,1
x 38 3,3
39 2,3
40 2,1
41 6,3
42 4,2
43 1,3
44 6,2
45 6,3
46 5,2
47 4,5
x 48 2,2
49 5,2
x 50 3,3

Good strategy to find experimental and theoretical probabilities.

$P(\text{pals}) = \frac{7}{50} = \frac{14}{100} = 14\%$

$P(\text{M's}) = \frac{43}{50} = \frac{86}{100} = 86\%$

Practitioner

The Double Dilemma

No. 1 Experimental For curiosity and to compare to Mrs. Cullens $\frac{1}{3}$ rolling doubles, I did my own trial. Here's is what I have

Joanna [redacted] Trial

50 trials

$P(\text{rolling doubles}) = \frac{7}{50} = \frac{14}{100} = 14\%$

$P(\text{rolling mixes}) = \frac{43}{50} = \frac{86}{100} = 86\%$

Mrs. Callens Game Results

$P(\text{rolling doobles}) = \frac{1}{3}$

$P(\text{rolling mixes}) = \frac{2}{3}$

Good math content language of probability notation, fractions and percents.

No. 2 Theoretical But what's theoretical? Here are the combinations

Mixes

1. 1 and 2
2. 1 and 3
3. 1 and 4
4. 1 and 5
5. 1 and 6
6. 2 and 1
7. 2 and 3
8. 2 and 4
9. 2 and 5
10. 2 and 6
11. 3 and 1
12. 3 and 2
13. 3 and 4
14. 3 and 5
15. 3 and 6
16. 4 and 1
17. 4 and 2
18. 4 and 3

Doubles

1. 1 and 1
2. 2 and 2
3. 3 and 3
4. 4 and 4
5. 5 and 5
6. 6 and 6

Good reasoning to include 1-3 and 3-1 as combinations.

Practitioner

19. 4 and 5
20. 4 and 6
21 5 and 1
22. 5 and 2
23. 5 and 3
24. 5 and 4
25. 5 and 6
26. 6 and 1
27. 6 and 2
28. 6 and 3
29. 6 and 4
30. 6 and 5

Thirty combinations!
and ther are only six combinations!
for doubles. So here's the theoretical probability.

P (doubles) = $\frac{6}{36} = \frac{1}{6} = 16\frac{2}{3}\%$
P (mixes) = $\frac{30}{36} = \frac{5}{6} = 83\frac{1}{3}\%$

So Mrs. Callens got <u>very</u> lucky. She
got doubles twice as much as
you would expect her to get.

Expert

1st	100
1	7-3
2	2-4
3	8-2
4	3-5
5	6-2
6	5-4
7	6-6*
8	3-1
9	2-5
10	4-3
11	3-1
12	5-3
13	5-6*
14	1-1*
15	2-1
16	24
17	4-3
18	6-1
19	5-1
20	4-5
21	1-4
22	1-3
23	4-3
24	3-2
25	5-4
26	6-6
27	6-5
28	2-5
29	5-6*
30	4-3
31	6-1
32	3-5
33	2-6
34	1-1*
35	4-4*
36	2-6
37	26
38	2-6
39	3-3*
40	6-1
41	1-3
42	2-1
43	1-3

44	6-2
45	4-4*
46	8-4
47	2-4
48	4-5
49	5-3
50	1-6
51	2-3
52	1-4
53	6 4
54	8 5
55	6-7
56	5-7
57	5-2
58	5-4
59	1-5
60	7-2
61	5-1
62	2-6
63	3-3*
64	4-2
65	5-6
66	4-5
67	2-1
68	5-6
69	5-1
70	6-1
71	2-5
72	4-2
73	1-1*
74	4-4
75	2-5
76	5-5*
77	3-5
78	4-2
79	5-7
80	5-2
81	4-2
82	5-4
83	6-3
84	3-3*
85	3-5
86	3-3*
87	5-1
88	4-3
89	2-3
90	1-2
91	6-5

92	4-3
93	6-2
94	5-6
95	6-7
96	2-6
97	2-2*
98	3-1
99	1-3
100	1-3

Doubles = 15%
almost $\frac{1}{6}$ of the time

Possible combinations for doubles = 6

Possible combinations for singles = 30
Total combinations = 36 $6 = \frac{1}{6}$ of 36
so I should expect to get doubles $\frac{1}{6}$ of the time

Expert

Double Dilemma

I solved this problem by first experimenting 15% of the time I got doubles and I saw that it was pretty close to $16\frac{2}{3}$ % of $\frac{1}{6}$ of the time. Then counted all the possible combinations fo two dice, and I found that there were 36 total. Of those 36, six were doubles, and six is exactly $\frac{1}{6}$ of 36, so theoretically I should have gotten doubles $16\frac{2}{3}$% of the time. $\frac{1}{6}$ is half of $\frac{1}{3}$, so I guess Mrs. Callens was just really lucky.
aother way to do this problem.

# of dice	total outcomes	all dice the same	proportions	percentage
1	$6 = 6^1$	6		
2	$6 \cdot 6 = 6^2$	6	$6/6^2$	$\frac{6 \cdot 100}{6^2} = 16.6\%$
3	$6 \cdot 6 \cdot 6 = 6^3$	6	$6/6^3$	$\frac{6 \cdot 100}{6^3} = 2.7\%$
4	$6 \cdot 6 \cdot 6 \cdot 6 = 6^4$	6	$6/6^4$	$\frac{6 \cdot 100}{6^4} \approx .463$
n	6^n	6	$6/6^n$	$\frac{6 \cdot 100}{6^n}$

Student uses complex reasoning to generalize problem to harder cases. Also, good use of math language.

This can also be made into a generalization.

Good observation.

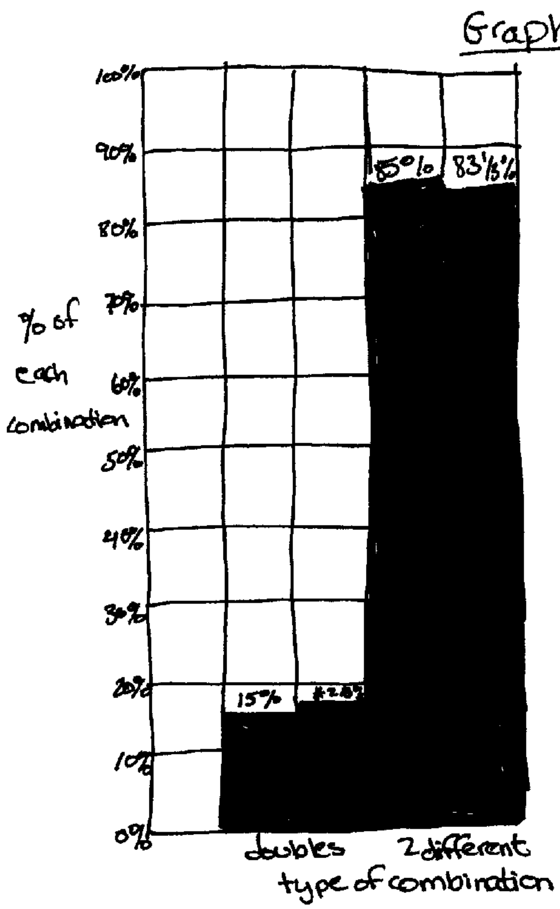
n = number of dice.
Generatization: $\frac{6 \cdot 100}{6^n}$ = percantage of the time the same number comes up on all the dice.

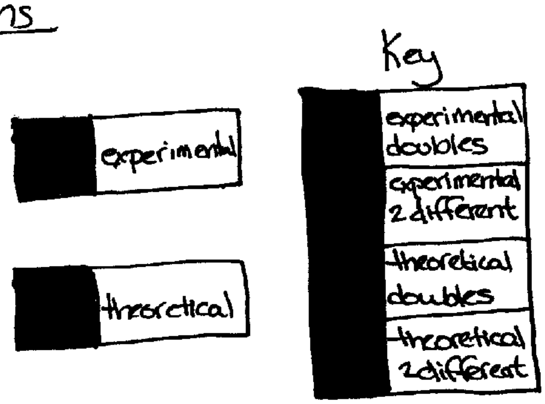
My generalization is the last step to system for solving this problem shown above. If you took, for example, 1 die, this is how my system would work. First you count how many combinations (or in this case, numbers) you can get with those dice/die. One way to do that is to multiply six times the number of dice. For any number of dice there will be six combinations where all the dice have the same number, so the proportion would be 6/6 for 1 and for n it would be

Expert

$6/6^n$. To get a percentage from that you multiply the numerator by 100 get $\frac{6 \cdot 100}{6^n}$. If you replace n with 1 then you would have $\frac{6 \cdot 100}{6}$ which equals when simplified, 100%, because since you have only one die, every time you role it, you're going to come up with only one number. This will work for any number of die. If you try counting out all the combinations without using my system, you'll get the same answer, but my way is much quicker, and mor efficient.

Expert





Outstanding use of representation.