Swinging Time

A metronome is a device used by musicians to time beats that change the rhythms of music. This is extremely helpful if someone wants to be trained in learning to produce the correct rhythms with a consistent beat.

Jody was loaned a metronome for a day and he did not want to spend the money necessary to buy his own. However, he had learned in science class that a pendulum swings at different rates of speed depending on its length. He determined that it might be possible to recreate the beat of the metronome by adjusting the length of the pendulum.

Jody has a 30 cm length of string. The number of beats are to be measured in 5 second intervals (i.e. 4 beats per 5 seconds). If the string is adjusted at 2 cm intervals, what are the possible beats that could be produced by the pendulum?

Jody was really excited about the number of beats that could be produced and decided to get a longer piece of string to produce more rhythms.

Based on an analysis of the data you collect from carrying out Jody's investigation, predict the slowest beat that could be produced by a 60 cm length of string.

Grade Levels 6 - 8

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Context

We were working on a series of tasks, which demonstrated exponential growth and had been creating graphs, including J-curves, which represented these numbers. In looking for a real-life situation where students could generate data to create a graph of interesting numbers, a friend and I, both math/science teachers, thought of this task. Students could apply what they knew about pendulums, gather data and examine the mathematical relevance of their results.

What This Task Accomplishes

Students will have the opportunity to demonstrate the ability to make charts, tables and graphs for investigative results. Students will have the opportunity to extrapolate data from their graph and the opportunity to evaluate the reasonableness of their solution.

What the Student Will Do

Students were asked to create pendulums given 30 cm of thread and washers. They needed to test the number of swings possible in five seconds with two cm intervals of thread. They were to predict from their findings how many swings could be expected with 60 cm of thread. They were expected to create some mathematical system for looking at their data in order to make a prediction for 60 cm.

Time Required for Task

60 minutes

The students were able to complete this task in one hour. Some of them worked beyond the hour of class time to complete the write up or to work on an extension idea.

Interdisciplinary Links

This was a perfect math/science interdisciplinary task. It would also have been a great opportunity to tie in technology with use of spreadsheets to record data and create graphs electronically. I did not do this because of lack of access and time constraints. The task also lends itself well to integration with music and dance.

Teaching Tips

I learned the hard way that having students hold the string leads to very inaccurate data collection. Some better ways would be to have them put a pushpin tack in a bulletin board and hang the thread by a loop from the pin to swing it. Students could also suspend a meter stick between two desks and tape the thread to the stick. Use thread instead of heavier string, as the size of some student's knots greatly affects the results. Use stopwatches if possible, although many students have good enough watches. Students have to work in pairs in order to time accurately. Even with all these tips, students will probably not get reliable enough data to be able to accurately predict the number of swings for 60 cm, but they should be able to graph these results and see a predictable curve develop. The problem is that most see a line and not a curve, predicting about one or two swings. It is interesting to see which students accept this prediction and which ones realize that it just does not make sense to expect one swing in five seconds. It helps students to realize that it is always good to verify your results with another method if possible and in this case, it is possible.

Suggested Materials

- Metal washers - I used some that were about one inch in diameter. All students should have ones the same size. Having other washers available of different sizes will suggest extension activities.
- Thread - I used button thread, which is more heavy duty than regular thread. We tried string, but the knots were too bulky and added excess mass to the washers.
- Stopwatches or a clock that ticks off seconds.
- Metronome for demonstration - although a metronome is not a perfect analogy for what the students are actually doing, as there is a spring mechanism in an actual metronome.
- Meter sticks
- Graph paper

Possible Solutions

The Expert exemplar has a correct solution.

Benchmark Descriptors

Novice
The Novices had a great time "playing", but did not get to the math involved. Many either did not complete a graph or did not number and/or label one correctly. They had no idea how many swings to predict with 60 cm. Some of them did get 60 cm of string and try it for themselves rather than make a prediction. They really applied little or no math to the exercise beyond measuring the intervals of thread. Their measurement skills for both lengths of string and time intervals were not completely reliable.

The exemplar student did gather and display data, but never answered the question asked. This student does not say what was done to solve the problem or why it was done.

Apprentice
These students all attempted to graph their results. Their data was rather unreliable as they were not very careful. Their graphs were unpredictable, as the points were all over the place. Rather than re-testing, they just declared their results unpredictable and quit. Most of these students had better graphing skills, but some of their intervals were incorrectly executed. Their strings tended to be measured accurately; their time intervals were slightly less accurate.

The exemplar student has the basic idea and makes a fair attempt at creating charts and a graph. The student does not have reliable enough data to see a pattern and does nothing to confirm the answer given.

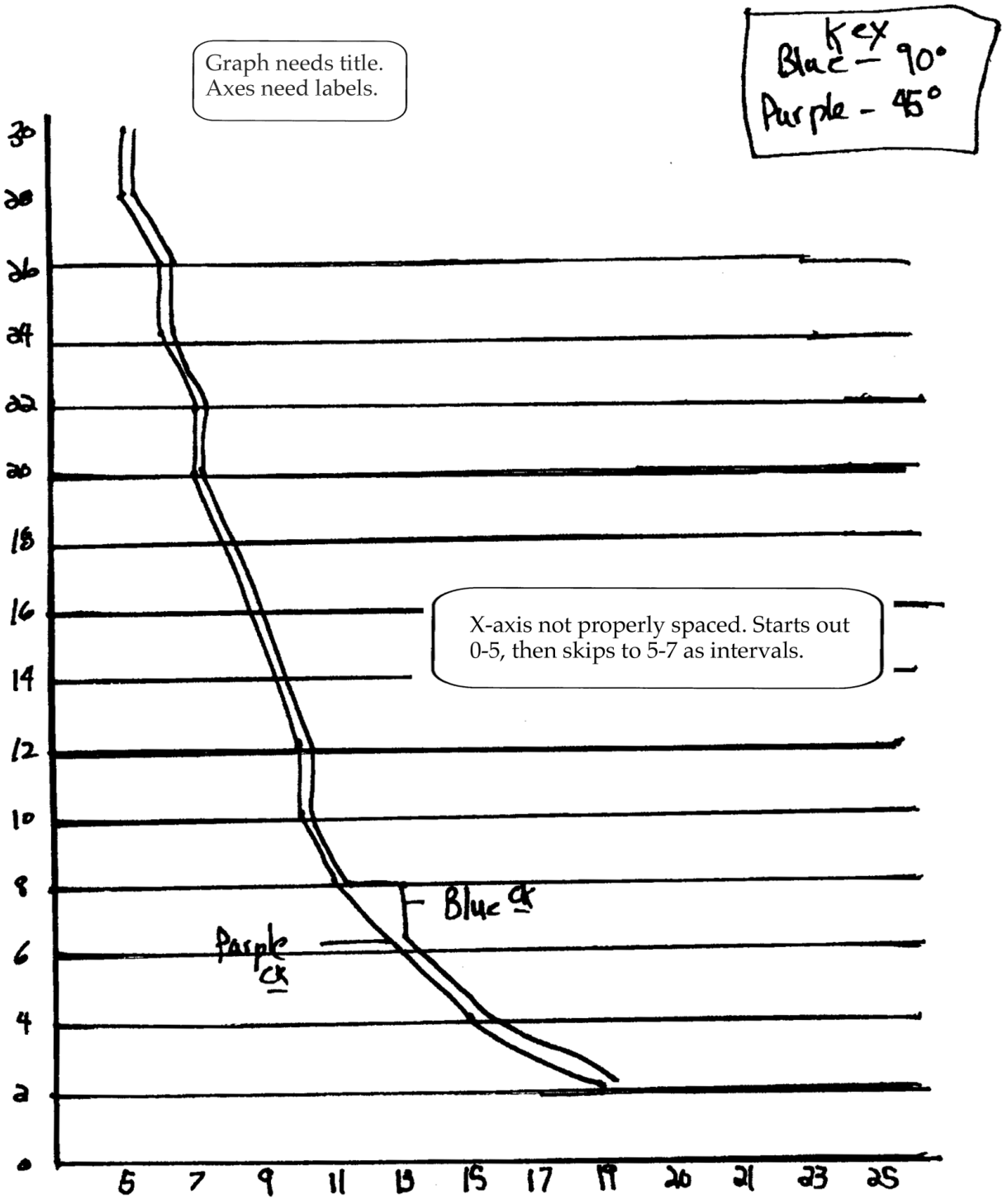
Practitioner
These students followed the directions accurately. They were careful in gathering and recording their data. They were able to extrapolate data from their graphs that indicated only one or two swings with 60 cm of string and they accepted that as fact and moved on. They did not question the validity of their data and did not extend their thinking on this task. Their tables and graphs were for the most part accurate and their solutions were quite well written. Their measurement of time and distance tended to be reliable.

The exemplar student uses good mathematical language and good representation. The student uses sound reasoning up to the end of their work, but stopped too soon.

Expert
These were the students who extrapolated their data, predicted one swing on 60 cm and realized that did not make sense. They continued by testing it out and accounting for the discrepancy. They conducted more than one trial and accounted for the differences recorded. Their charts and graphs were accurate. Many of them went on to extend the investigation with other sized washers or by using multiple washers to increase the weight. They found methods to stabilize the swinging washers as well or at least acknowledged that this variable was affecting their results.

I selected this piece as the exemplar for its clear, concise content. The student uses good mathematical language to state what was done to solve the problem and good representation to demonstrate the findings. The student shows excellent reasoning in recognizing the impossibility of the most obvious pattern. The student uses a second method to solve the problem, compares the results and goes with the solution that makes the most sense to the student.

Novice



Novice

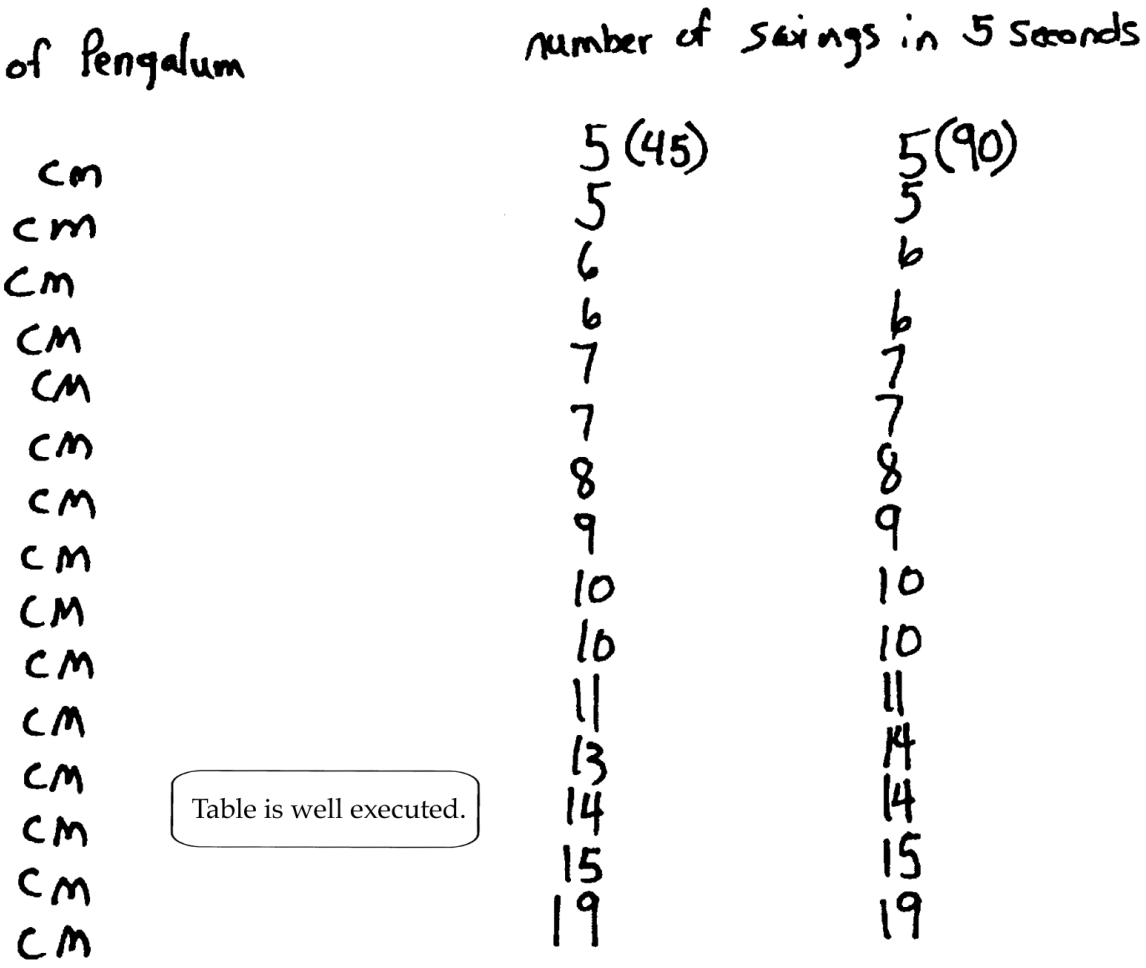


Table is well executed.

Apprentice

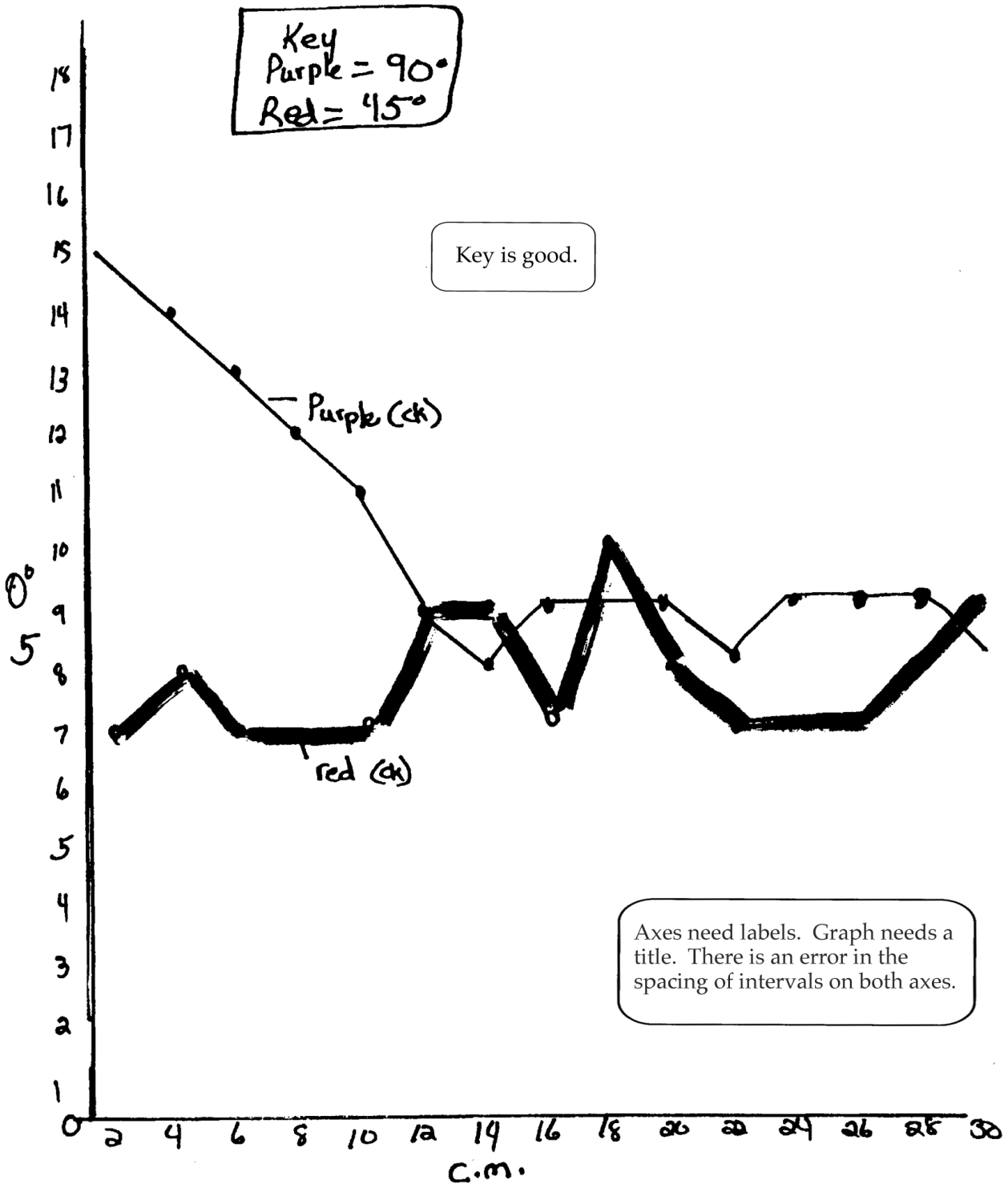
I Solve my problem by making a graph and 2 charts, First I made one of my charts and I had colums that had cm., 45° and 90°. On the cm. colum I went down from 2's and started at 30 and ended at 2. I had to see how many Swings a rope with a washer tied on the end. would swing in 5 second's from 30cm to 2cm. Then I did the same thing agian but I Swong it from 90°. Then I did a second tral and graphed it out. After that I had to predict how many swings it would be from 60cm to 32cm. from my graph but my graph had no pattrens or connections. So I just made another chart and did it from 60cm all the way down to 32cm. My answer I got at 60cm was (5) swings. And at 30cm I only got 7. I found out that the shorter the string is the more it swings. At 90° swings my graph straled to slop down then it slarted to go up and down. But at 45° I had no pattrens, at all.

The problem was not solved by making charts and a graph. These only organize and display the data gathered.

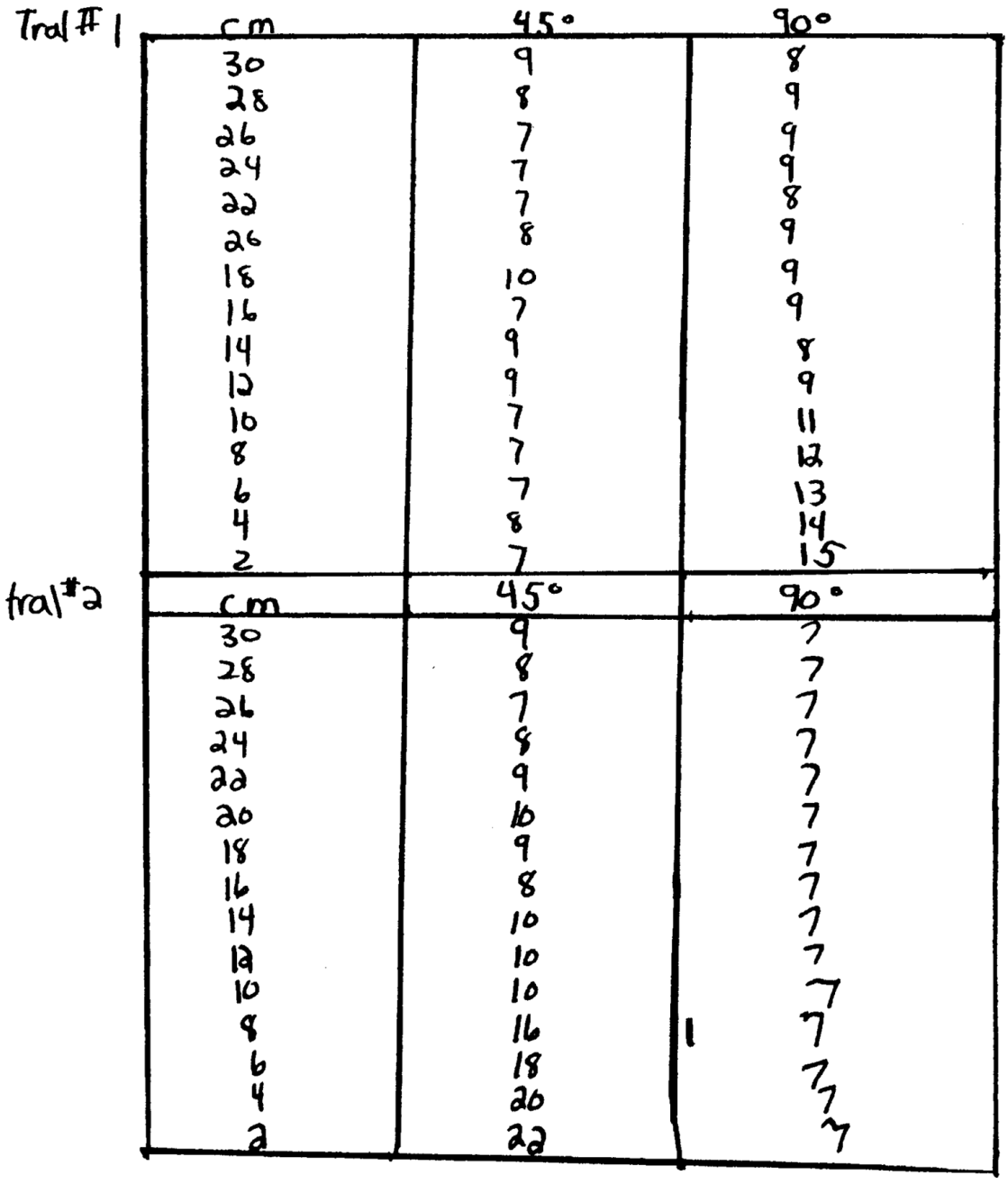
The description of the chart is unnecessary as the reader can get this information by just looking at the chart.

Student makes no observation as to why the slopes are as they are, and what information is being revealed by the graph.

Apprentice



Apprentice



Trial # 1

cm	45°	90°
30	9	8
28	8	9
26	7	9
24	7	9
22	7	8
26	8	9
18	10	9
16	7	9
14	9	8
12	9	9
10	7	11
8	7	12
6	7	13
4	8	14
2	7	15

trial #2

cm	45°	90°
30	9	7
28	8	7
26	7	7
24	8	7
22	9	7
20	10	7
18	9	7
16	8	7
14	10	7
12	10	7
10	10	7
8	16	7
6	18	7
4	20	7
2	22	7

A better table would have space for both trials without copying the string length again. It would make it easier to compare results from trials using same length strings.

Apprentice

cm	swim 5 sc. 90°
60	6
58	7
56	6
54	6
52	6
50	5
48	6
46	4
44	6
42	7
40	7
38	7
36	7
34	8
32	7

Should have gathered data from more than one trial.

Practitioner

Makes an important observation here. Students were surprised that the angle of the drop tended to make little difference in the number of swings.

Portfolio Write-Up

I was asked by my math teacher, Mrs. Kenny, to find out how many times a washer on a string, known as a pendulum, would swing back and forth at intervals of 2 cm. in 5 seconds. The length of string was predetermined at 30 cm. in length.

I used my watch to time myself for the 5 seconds. I had already marked 2 cm. intervals on my string. I swung each different interval 3 times and took the mode of each of them. At first I dropped the washer from a 45 degree angle and then from a 90 degree angle. I used a table to keep track of my data. I counted one swing as over and back.

It was very interesting to watch the table progress. The number of swings remained about the same between the 45 degree drop and the 90 degree drop. I was very interested to find out that they had remained about the same. I thought that the number of swings would be greater from the 90 degree drop because there was more power behind it.

Please See Attached Table

Then I took my data I had collected on my table and made a line graph. The X axis had the length of the pendulum on it and the Y axis had the number of swings on it. The color of the line for the 45 degree angle is red and the color of the 90 degree line is blue. The red and blue lines criss-crossed across each other at some points and at others they were the same.

Please See Attached Graph

I was also asked to estimate the number of swings at 60 cm. by extrapolating from my graph. My line graph starts up high, then slopes down and levels off. The slope is very interesting. It reminded me of the slope I had on one of my other graphs, except backwards. The conclusion that at 60 cm, the pendulum would swing 5 times in 5 seconds. Then I made a 60 cm pendulum and tried it. The pendulum swung three times in 5 seconds. So, according to my extrapolating the pendulum would swing 5 times in 5 seconds. But from actually trying it, I concluded that it swings 3 times in 5 seconds.

Student does not state basis for this conclusion.

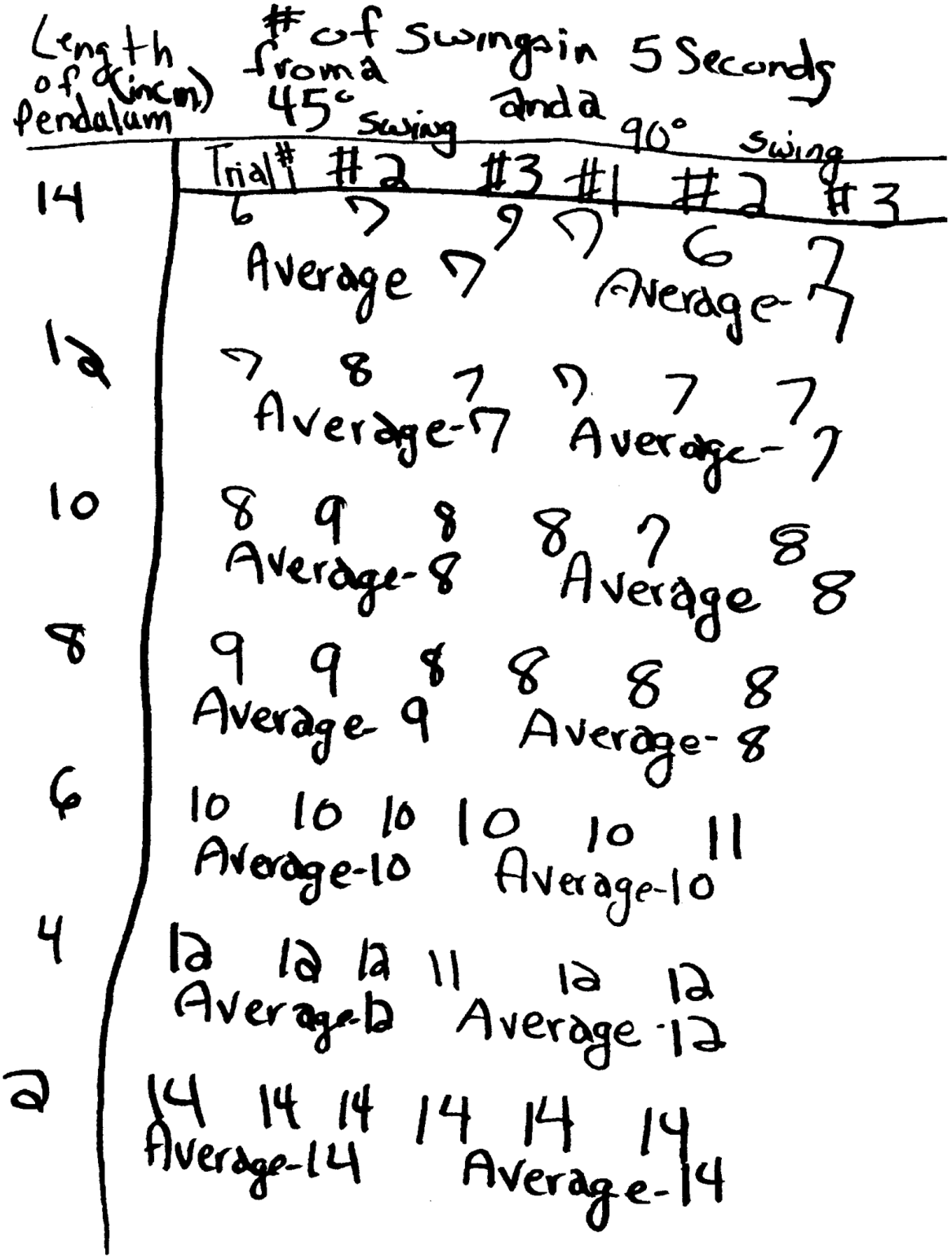
Student stopped short of attempting to explain the discrepancy between the predicted and actual # of swings.

Practitioner

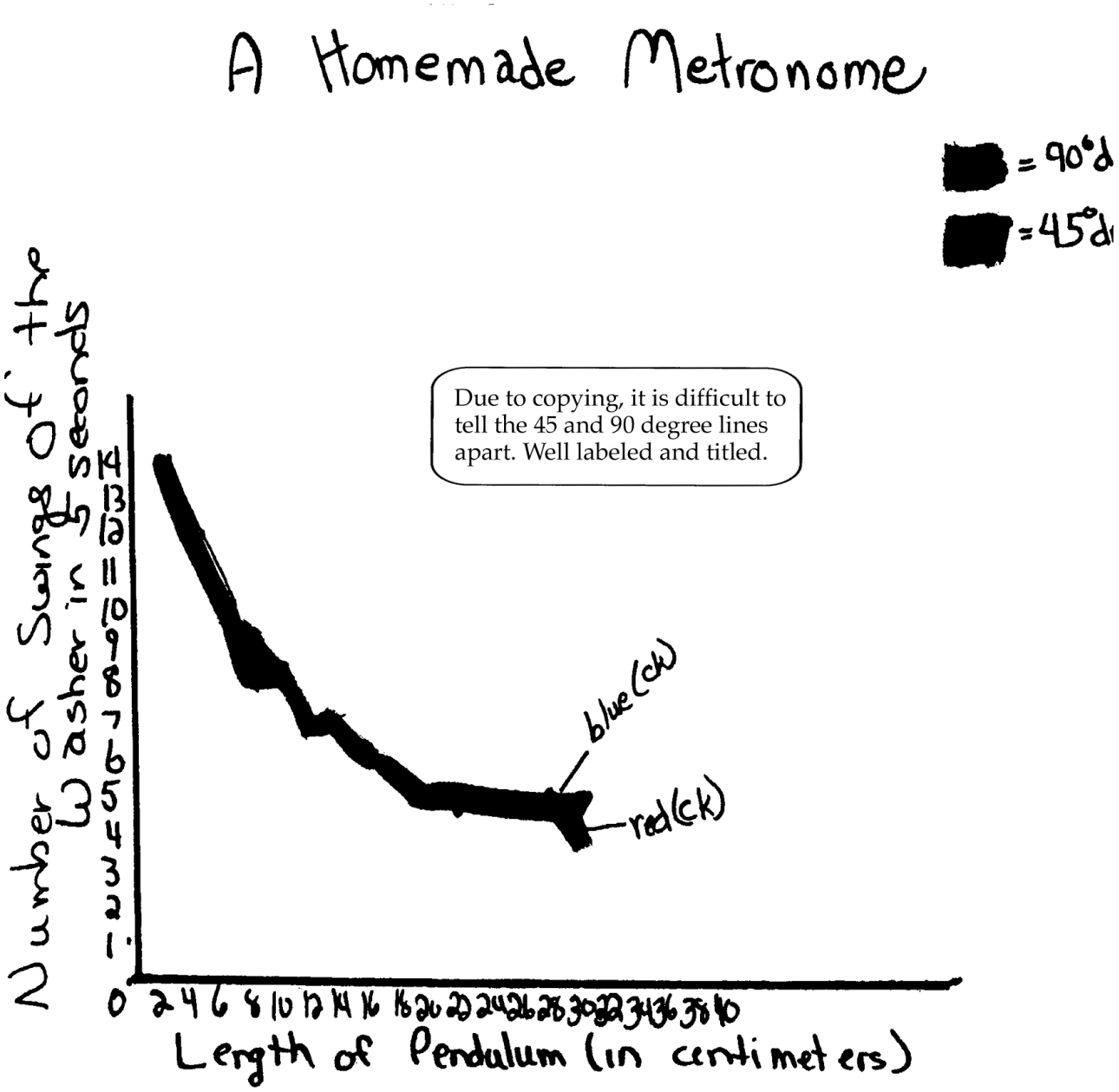
Table of Swings

Length of Pendulum (in cm.)	# of swings in 5 seconds from a 45° Swing			and a 90° Swing		
	Trial #1	#2	#3	Trial #1	#2	#3
30	4	4	5	4	5	5
	Average - 4			Average - 5		
28	5	5	5	5	4	5
	Average - 5			Average - 5		
26	5	5	5	5	5	5
	Average - 5			Average - 5		
24	5	5	5	5	4	
	Average - 5			Average - 5		
22	5	5	5	5	5	4
	Average - 5			Average - 5		
20	6	5	5	5	5	5
	Average - 5			Average - 5		
18	6	6	6	6	6	6
	Average - 6			Average - 6		
16	6	6	6	6	6	6
	Average - 6			Average - 6		

Practitioner



Practitioner



Expert

Shows excellent reasoning in recognizing the impossibility of the most obvious pattern.

The student states clear understanding of the task.

A Swinging Time

In this task, I was asked to extrapolate the lowest number of beats that could be produced by a washer with a 60cm length of string (not including the knot) which is called a pendulum. The beats are measured in five second intervals. To get the data to solve the task I had to use a 30cm length and measure the beats adjusted at 2cm intervals.

The first thing I did was to make a table to record my data in an organized fashion. On the table I have data for every 2cm interval between 30cm and 2cm, and the data for 60cm. Since the task did not say so the teacher said to drop the washer at a 45° angle and a 90°. I measured the beats (a beat is a swing from one side to the other) for each angle twice, to make sure my data was correct. My data seemed consistent and easily predictable. So, to show my data I made a line graph, the 90° drop in red and the 45° drop in grey. The *x* axis is the length of the string. The *y* axis is the beats per 5 sec. interval.

So, continuing the pattern on the graph, there are two beats for a 45° drop and one beat for a 90° drop. This doesn't make sense, because it would be impossible for one or two beats to last for a whole five seconds with only 60cm of string. Also, according to the graph, in a few more centimeters there would be zero swings, and a few more intervals after that there would be negative numbers. This, of course, is impossible, because no matter what the size or weight of the pendulum, it will always swing. So, I would say that the pattern somewhere past thirty centimeters is not at all significant, because somewhere, the pattern will even out, and two centimeters won't make much of a difference.

To solve the problem using a different method I got a 60cm string and a washer and tried it. When I did this, I got six beats in five seconds. This answer, in comparison, is obviously, much more accurate than the graph's.

Uses a second method to solve the problem and compares results.

Uses good math language to concisely state what was done to solve the problem.

Expert

Length of string # of Swings /5 sec.

	45° swing		90° swing	
	Try 1	Try 2	Try 1	Try 2
30cm	10	10	9	9
28cm	10	10	9	9
26cm	11	11	10	10
24cm	11	11	10	10
22cm	12	12	11	11
20 cm	12	12	11	11
18 cm	13	13	12	12
16 cm	13	13	12	12
14 cm	14	14	13	13
12 cm	14	14	13	13
10cm	15	15	14	14
8 cm	15	15	14	14
6 cm	16	16	16	15
4 cm	16	16	15	15
2 cm	17	17	16	16
60 cm	6			

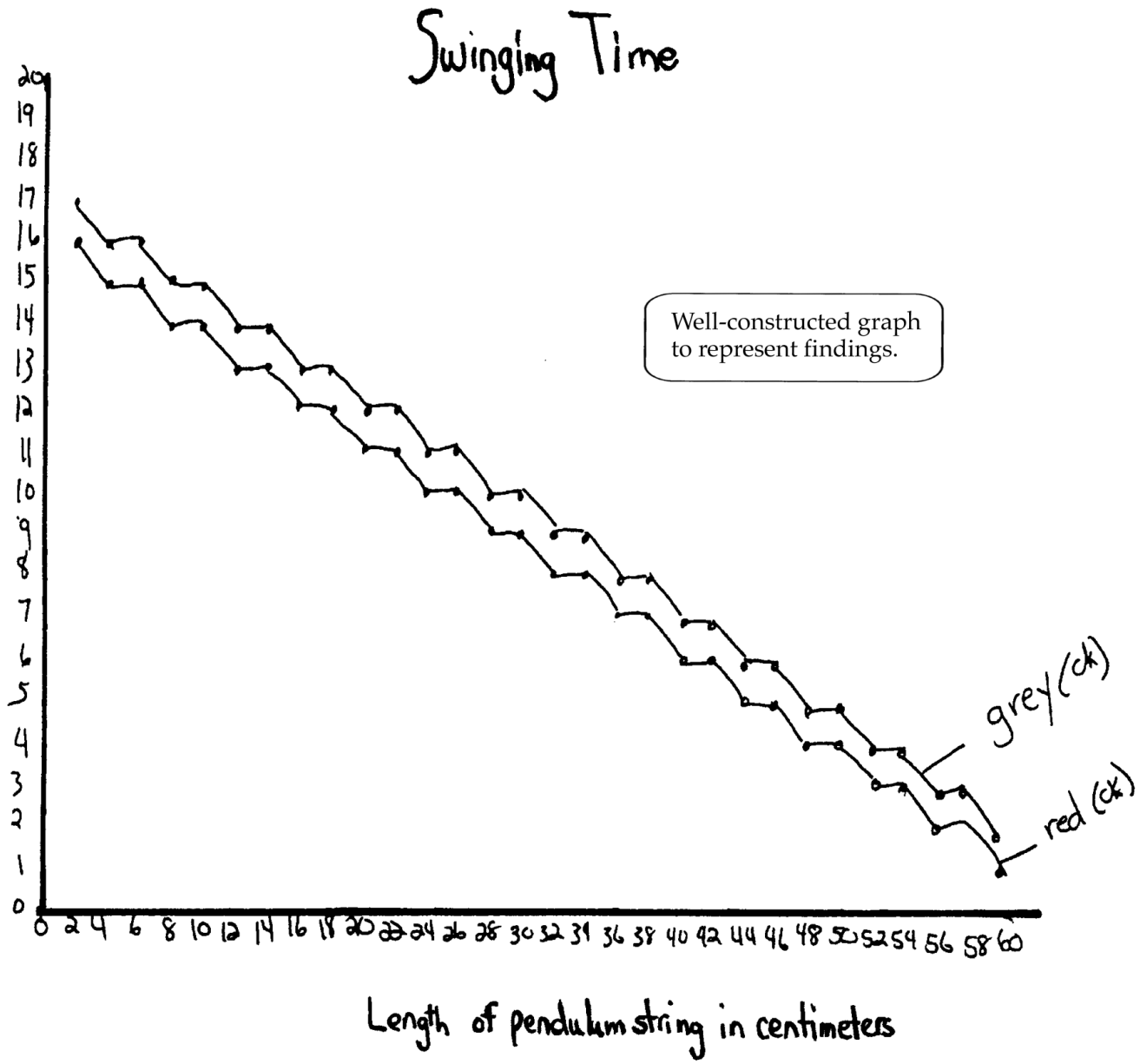
Well organized, labeled and title table for organizing data.

Expert

Pendulum Swings

Length of string	# of Swings/5sec			
	45° swing		90° swing	
	Try 1	Try 2	Try 1	Try 2
30cm	10	10	9	9
28cm	10	10	9	9
26cm	11	11	10	10
24 cm	11	11	10	10
22 cm	12	12	11	11
20 cm	12	12	11	11
18 cm	13	13	12	12
16 cm	13	13	12	12
14 cm	14	14	13	13
12 cm	14	14	13	13
10 cm	15	15	14	14
8 cm	15	15	14	14
6 cm	16	16	15	15
4 cm	16	16	15	15
2 cm	17	17	16	16

Expert



Swinging Time