Building Block Dilemma

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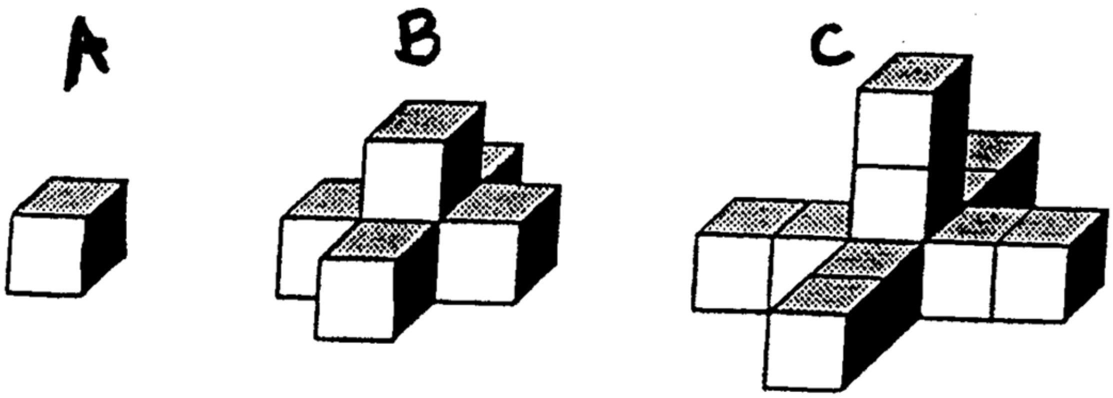

Grade Levels 6 - 8

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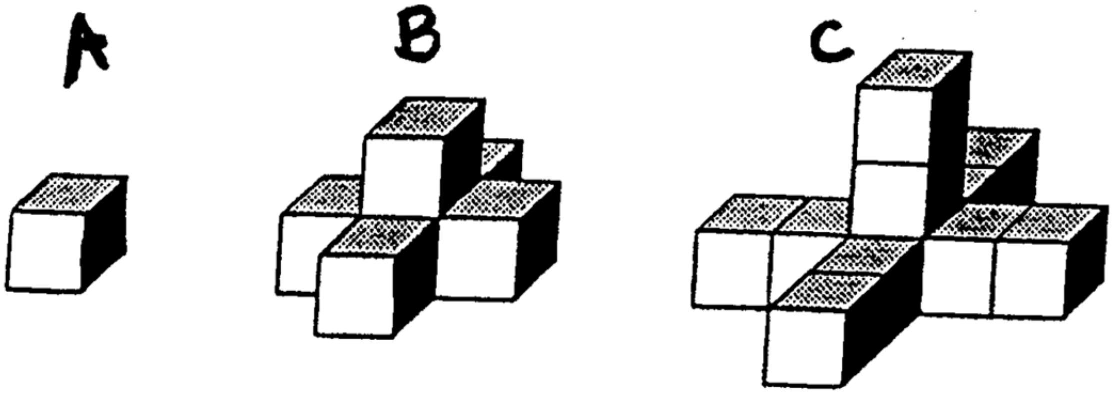


Context

My sixth grade class just finished a unit on integers and graphing simple equations. The class was able to predict where a certain equation would be located on a coordinate grid. We worked with equations like x + 4 = y, (-x) + 4 = y, x + (-4) = y and (-x) + (-4) = y. The graphing unit came out of my desire to work with integers - to show them that you could see equations on graphs and that graphs help to find other solutions to equations. I wanted to continue with the idea that an equation is really describing a pattern. Some patterns from equations can be graphed and some equations can come from building blocks. Sixth grade is a little early for this, but I knew that all of my students would be able to access this problem and those that were ready would reach a generalization.

What This Task Accomplishes

This task makes the connection for some students between a physical pattern and an equation or numerical pattern.

What the Student Will Do

Some students asked for blocks to construct the buildings. Others asked for graph paper to construct the next pattern. Some students made a chart of cubes in each tower and cubes in the whole building to look for a pattern. Other students could visualize how the structure was growing and could almost come up with a generalization from that information.

Time Required for Task

45 minutes

Most of the students investigated and collected their data in a 45-minute period. Some needed a few more minutes to pull it all together.

Interdisciplinary Links

Some links could be the study of patterns in nature or in man-made objects.

Teaching Tips

Many of my students asked about the block that you cannot see in the towers. They wanted to be sure there was a cube there. None of my students asked for blocks, but I had them available in case a student felt s/he wanted to build the structures.

Suggested Materials

- Blocks
- Paper
- Pencil
- Graph paper

Possible Solutions

A five-block high tower takes 21 blocks and a 10-block high tower takes 46 blocks (five blocks are added each time - one to the tower and one to each of the four supports). Some observations that lead to a generalization are that there is one more block in the tower than the four supports. So if *h* = height of tower then...5h - 4 = total blocks. You need to subtract four because each support has one less block than the height of the tower. Another way to think of this is...4(h -1) + h = total blocks.

Benchmark Descriptors

Novice
A Novice student will not be able to become engaged in the problem. Other Novices, as in the benchmark, may seem to understand that they need to find the total blocks, but their numbers make little sense and there is no explanation as to how they came up with their numbers.

Apprentice

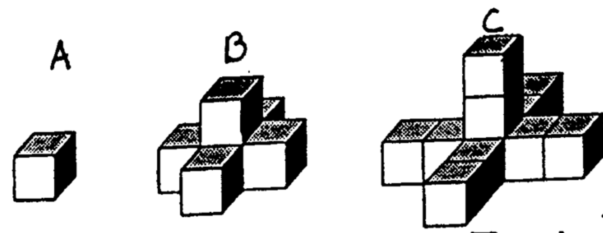
Apprentices will have a good approach. They may have a chart of the number of blocks in the tower and the total blocks in the structure, but will make a mathematical error in their calculations. They may also make a drawing that is not complete.

Practitioner
Practitioners will be able to show the towers growing by one block and the total number of blocks growing by five. They will be able to find the number of blocks for a five and 10-block tower. They should be able to see the pattern that the tower increases by one block each time and that the total blocks increases by five. They should have a chart or a diagram of the towers.

Expert
Experts will be able to find the solution to the five and 10-block high towers, but will also be able to generalize the solution. Experts will solve the problem two ways to verify their solutions. A chart will let them see a pattern that the total blocks increase by five each time. They should also be able to see the relationship between the tower and the supports. They should be able to explain their equation using the diagram of the tower.

Novice

H = Hieght
~~B = Base~~
T = Tower



* There is a block in the center of the base

Letter	height	blocks in all
A	1	1
B	2	5
C	3	9

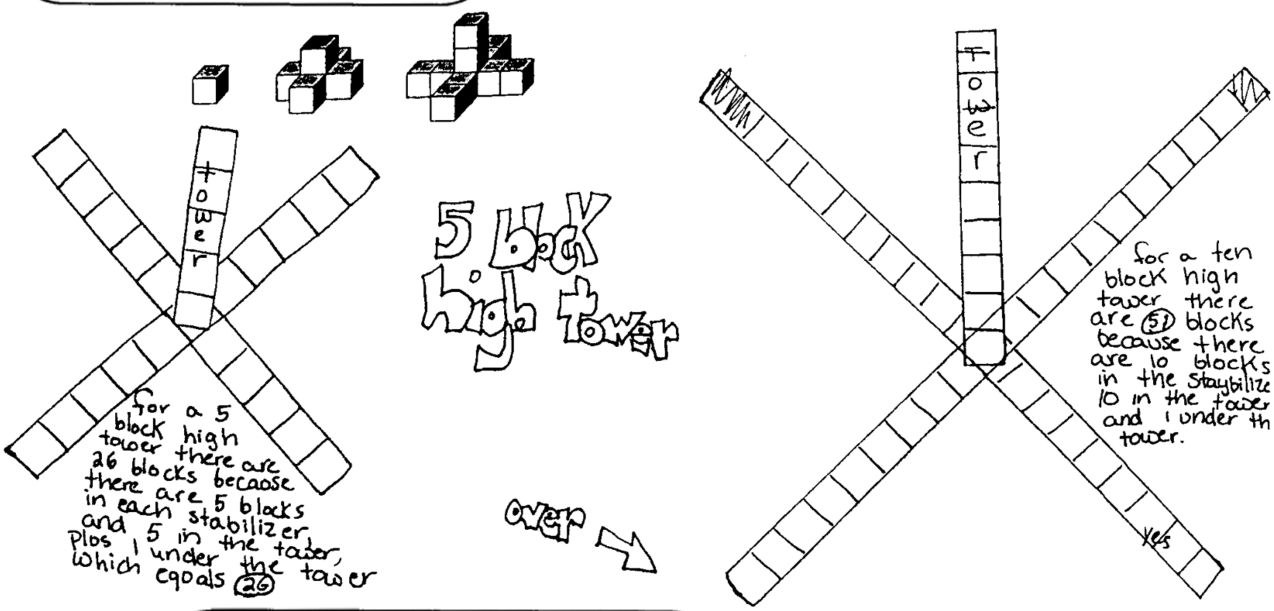
First I made a table with the the tower its hight and it base. The I wrote down H. and thought of ways to get from H to B. When I found 1 that worked I wrote it down. Next I applied it to the tower charts in my head

This chart is confusing and does not continue to a 5 and 10 block high tower.

The student is doing something in his/her head, but cannot explain any reasoning.

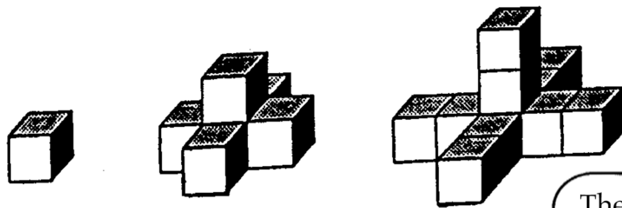
Apprentice

The student used a graph paper cut-out to draw a 3-dimensional drawing of each tower.
The drawings help to explain the error in the solution.



The student has an error in his/her thinking about the number of cubes in the tower and the number of cubes in the stabilizers. I have confidence that if the student was asked to look over the solution, they could fix the error.

Practitioner



Blocks of	Total
1	1
2	6
3	4
4	16
5	21
6	26
7	31
8	36
9	41
10	46

The student made a good observation giving evidence that s/he understood the problem and could see the pattern in the list of numbers.

EACh time the height rises, the total amount increases +5 minus 5 = 21 · 10 = 46

This observation gives evidence that the student could connect the pattern in the list of numbers with the visual drawings.

NB: It expands on top and on the bottom

Expert

(1.)

$(4[V-1])+V=A$

* 4 = the four sides on which the horizontal blocks are placed;
V = the vertical blocks (including the base block);
1 = the base block (which is subtracted because I don't want to include it in the horizontal blocks)
$(4[V-1])$ = horizontal blocks
A = Answer

eg:
$(4[5-1])+5=21$ blocks

The student makes the observation that the blocks increase by 5. The student comes up with a second formula, this is his/her third strategy!

The student makes the algebraic equation in terms of the blocks. The student tests the formula to make sure it works.

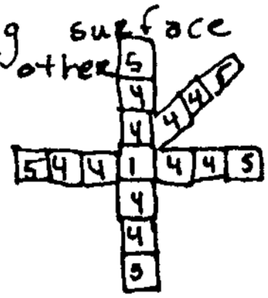
Vertical	1	2	3	4	5	6	7	8	9	10
Horizontal	0	4	8	12	16	20	24	28	32	36
Total	1	6	11	16	21	26	31	36	41	46

(2.)

Observation: The total number of blocks in each tower w a different number of vertical blocks increases by 5 with each block added vertically. Little like a sequence

(3.) $1+[V-1]5=A$
Another version of the first equation

Calculating surface of blocks exposed (not touching other)



$\boxed{6}$

$\boxed{26}$

26

$4\cdot[5(V-1)]$

16
66

$26+[([V-2]\cdot 5)4]$

The student has taken the problem and made a harder problem. S/he found a formula to find the surface area of each structure.