Baffling Bacteria

We have studied bacteria and know that they reproduce by fission or simple cell division. Streptococcus bacteria cause the well-known illness, strep throat. The bacteria can reproduce every 20 minutes.

According to a published source, if these bacteria continue unchecked, at the end of 24 hours this colony of bacteria would have a mass of 2 million pounds! Needless to say, this could not really happen in your throat.

If you inhaled a single strep bacterium today at noon and it reproduced, unchecked, every 20 minutes, at approximately what time would the colony outweigh you?

Grade Levels 6 - 8

Baffling Bacteria

We have studied bacteria and know that they reproduce by fission or simple cell division. Streptococcus bacteria cause the well-known illness, strep throat. The bacteria can reproduce every 20 minutes.

According to a published source, if these bacteria continue unchecked, at the end of 24 hours this colony of bacteria would have a mass of 2 million pounds! Needless to say, this could not really happen in your throat.

If you inhaled a single strep bacterium today at noon and it reproduced, unchecked, every 20 minutes, at approximately what time would the colony outweigh you?

Context

We had been studying exponential growth in math and monerans in science. I wanted to connect these two concepts in the students' minds. In reading a reference book on bacteria, I came across the weight statistic in the task. I thought that students would be particularly engaged in this task if it related to them personally, which is why I wrote the question as I did.

What This Task Accomplishes

This task connects two curricular areas: it encourages students to recognize exponential growth and to use the powers of two to find a solution.

What the Student Will Do

Students will use their knowledge of the powers of two or binary numbers, to solve this task once they discover that they are working with numbers too large for their calculators. They will eventually discover that the colony of bacteria weighing 2,000,000 pounds at the end of 24 hours will weigh 1,000,000 pounds 20 minutes earlier. They can then work backwards until they approach their own weight.

Time Required

45 minute

However, some of the students worked beyond that time to word process their write-up and use computer programs to create graphs and tables of their results.

Teaching Tips

If students have not been taught binary numbers, this task may well be too frustrating for them. I believe it is a good test of their comprehension of this concept once they have been taught it. Many seventh grade students study bacteria in life science classes, so you might want to coordinate with the Science teacher on this task. The *NCTM Addenda Series on Patterns and Functions* has a series of lessons on exponential growth that I used prior to this task. You might want to take a look at that resource.

I also realized that some of my less-able students would have a very difficult time with this task, so I wrote an alternative task for those with fewer abilities. I gave this out to those students who expressed high levels of frustration after an initial investigation. All of the *Exemplars* work is in response to the original task. I have included the modified task to give you an idea of one way to adapt a task to ensure equity of access for all students.

Baffling Bacteria - modified

The bacteria that cause strep throat can reproduce by fission every 20 minutes. If you inhale a strep bacterium at noon today, how many will have been produced by the time you go home at 2:00 o'clock? How many will there be by 6:00 o'clock tonight? How many at midnight? Your calculator can not calculate the number that will have been produced by noon tomorrow so can you find some way to express this number without the aid of the calculator?

Suggested Materials

Scientific calculators

Possible Solutions

The exact solution depends on the weight of the student. All my students would be outweighed between 7:00 am and 7:20 am the next morning.

Benchmark Descriptors

Novice
The students in this group missed the concept of doubling and exponential growth. They may have had the concept of the time slots, but that was generally it. Many of them did have some mathematical representation that they had used in an effort to find a solution. The exemplar student understood about the 20-minute time blocks required for the bacteria to double. The student confused 20,000 with 2,000,000 as the number of bacteria at the end of 24 hours. Beyond that, the student did not understand the concept of doubling as evidenced by going "down by 1,000s".

Apprentice
There were a number of reasons why a student fell into the Apprentice category. Many of them simply made a number of calculation errors due to the fact that their calculators were not scientific and could not handle the size of the numbers. They were not able to recognize the errors of their thinking or did not recognize the exponential growth factor, so did not question

their inaccurate solutions.

The exemplar in this group was interesting to me, as I believe that this student understood the task and was able to get an accurate answer, although the student only "thinks" the solution is correct. It seems evident that either this student was coached in making the growth chart or was simply trying to regale us with great wisdom in trying to explain the rule of exponential growth - which is inaccurate and not followed. The graph is also rather interesting as it shows us nothing of importance in understanding the solution and was not used in finding the solution. The hand done chart seems to be what the student used to find the solution (in class) and is accurate. In this case the computer work detracted from the student's solution.

Practitioner
These students all tended to understand the task, but made mistakes in their calculations or had errors in reasoning that prevented them from getting correct solutions. The exemplar piece represents some good logic in dealing with numbers too large for the calculator. By a careless error, the calculations were all off by one 20-minute period, so the solution of 6:50 is inaccurate as is evident in the accompanying table. This student has some interesting mathematically relevant comments as well.

Expert
These students immediately recognized the connection between cell division and exponential growth. They began working backward at once and frankly solved the problem in record time. The challenge for them was to use good mathematical language and representation in describing their solutions. The exemplar of this group verified the result through calculating a 61-pound colony's growth from 7:00 am until noon to see if the result would be the two million pounds stated in the task. The result was very close. While this student had a brief write up, there was strong mathematical language and a good table showing the resulting doubling of the colony of bacteria.

Novice

Baffling Bacteria
Task write-up

I solved this problem by writing a list of times that were only 20 minutes long. The time changes every 20 minutes because thats when the bacteria reproduces by fission. I started at 12:00 noon till 12:00 AM! At the end of 24 hours theres 20,000. poonds of bacteria! So I had to count down from 20,000 by 1000's then 100's then 1,000's and ended up with 17,188825728 Pounds of bacteria. I solved this problem without the aid of the calculater.

I am not sure why it is a good thing not to use a calculator!

Counting down by 1,000's shows misunderstanding of doubling.

Confuses 20,000 with 2,000,000 lbs.

Novice

Batt ling Bacteria

At the end of 24 hours there is 20,000 pounds of bacteria!

At 12:00 - 1,718,825,728 4:00 - 500

At 12:20 - 9,869,934,59 4:20 - 6:00

At 12:40 - 68,735,538,146 4:40 - 7:00

At 1:00 - 68,719,476,740 5:00 - 8:00

At 1:20 - 44,920,487,936 5:20 - 900

At 1:40 - 2 XX 36 = 7:40 am 5:40 = 1,000

I am confused as to what is happening here. What is 24x36=7:40.

At 2:00 - 57 (19, 3) - 59 = 6:00 — 2,000

At 2:20 - 2^9 512 6:20 - 3,000

Where did 2^9 come from?

At 2:40 - 2^{58} 6:40 - 4,000

At 3:00 - 200 7:00 - 5,000

At 3:20 - 300 7:20 - 6,000

Novice

At 8:00 - 8,000

At 8:20 - 9,000

At 8:40 - 10,000

At 9:00 - 11,000

At 9:20 - 12,000

At 9:40 - 13,000

At 10:00 - 14,000

At 10:20 - 15,000

At 10:40 - 16,000

At 11:00 - 17,000

At 11:20 - 18,000

At 11:40 - 19,000

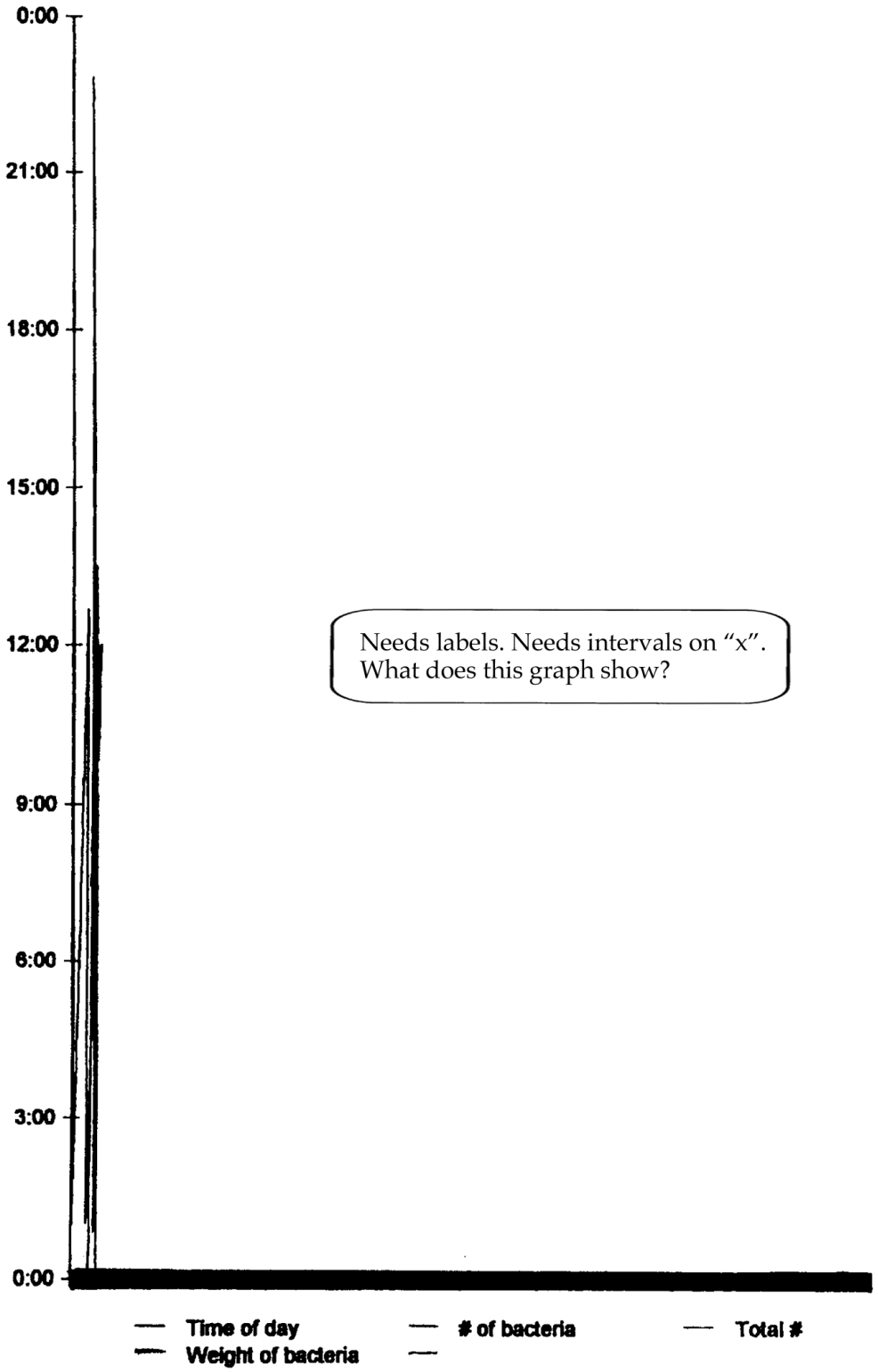
At 12:00 - 20,000

Apprentice

This is a write-up on the math portfolio piece Baffling Bacteria. The problem was this- if the virus multiplied 72 times in 24 hours and weighed 2000000lbs at the end of the 24 hours at what time would the colony outweigh you? First I took a sheet of paper and worked backwards from 2000000 at 12:00pm to 1000000 at 11:40 and so forth. Then I typed it up on the computer to see it better. Looking at the information I'd gathered I formed a general rule: the sum of the powers of 2 up to n is 2 raised to the power of n + 1 then minus 1 = the total amount of the supposed bacteria in your throat. Like, if you had 2 to the 5th and you wanted to know what the total was? You would take 32(2 to the 5th) times it by 2, and so raising the power from 2 to the 5th to 2 to the 6th, and subtracting 1. So in the end what you have is the total amount of bacteria when they have reproduced 5 times is 63. Because 2 raised to the 5th is 32, 1 + 2 + 4 + 8 + 16 + 32= 63 and 2 raised to the 6th is 64 - 1 = 63. Then I found the time at which the chart showed it corresponds with my weight.(7:20) Then I made a graph showing how the bacteria got progressively heavier. I conclude with the analysis that working forwards from 12:00pm to 12:00pm the next day I think at approximately 7:20 the bacteria will equal my weight.

The general rule is very confusing and not appropriate to the task.

Apprentice



Apprentice

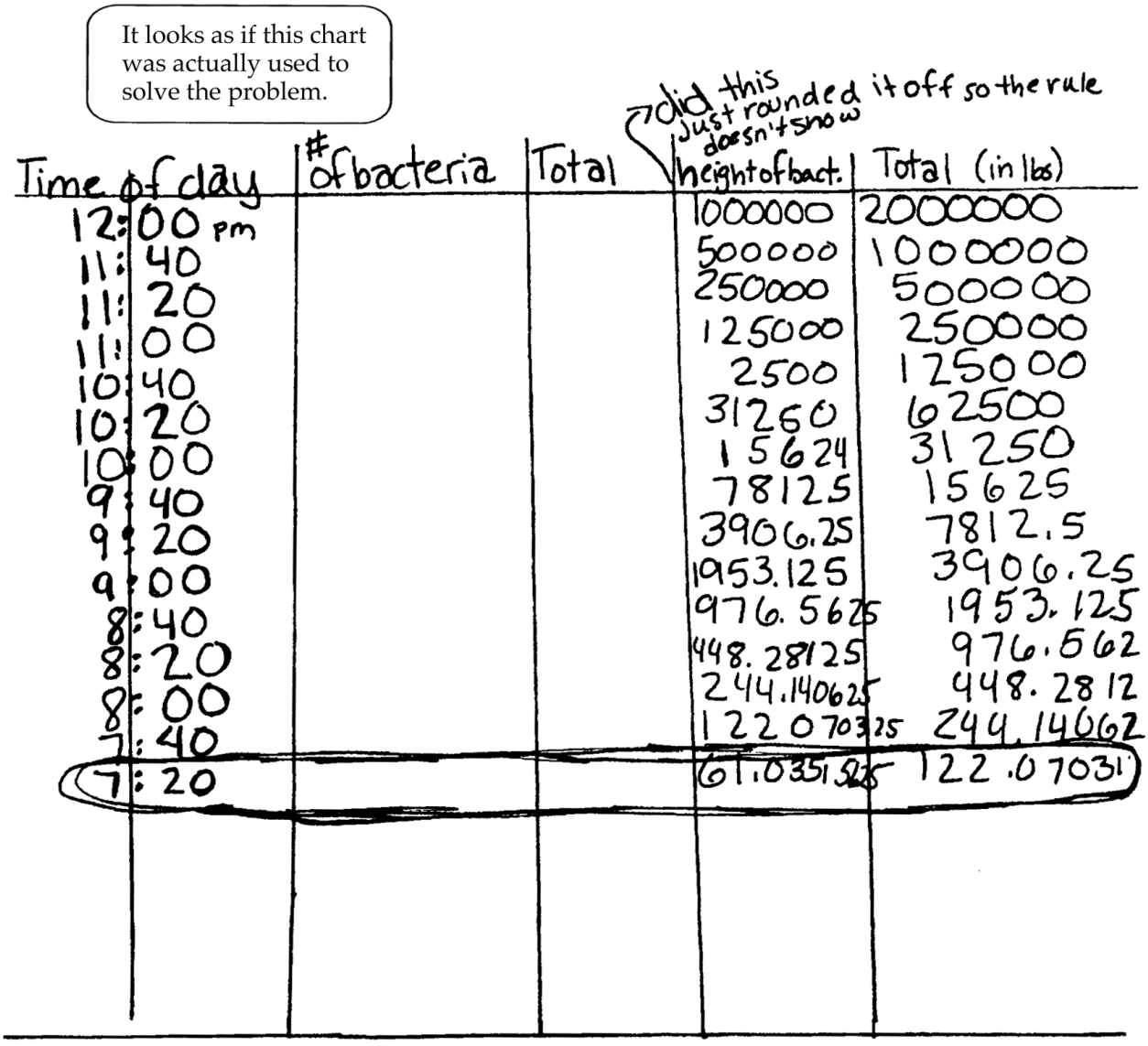
Time of day	# of bacteria	Total #	Weight of bacteria (in pounds)	
12:00	1	1	2.11758E-16	2.11758E-16
12:20	2	3	4.23516E-16	6.35275E-16
12:40	4	7	8.47033E-16	1.48231E-15
1:00	8	15	1.69407E-15	3.17637E-15
1:20	16	31	3.38813E-15	6.56451E-15
1:40	32	63	6.77626E-15	1.33408E-14
2:00	64	127	1.35525E-14	2.68933E-14
2:20	128	255	2.71051E-14	5.39984E-14
2:40	256	511	5.42101E-14	1.08208E-13
3:00	512	1023	1.0842E-13	2.16629E-13
3:20	1024	2047	2.1684E-13	4.33469E-13
3:40	2048	4095	4.33681E-13	8.6715E-13
4:00	4096	8191	8.67362E-13	1.73451E-12
4:20	8192	16383	1.73472E-12	3.46924E-12
4:40	16384	32767	3.46945E-12	6.93868E-12
5:00	32768	65535	6.93889E-12	1.38776E-11
5:20	65536	131071	1.38778E-11	2.77554E-11
5:40	131072	262143	2.77556E-11	5.55109E-11
6:00	262144	524287	5.55112E-11	1.11022E-10
6:20	524288	1048575	1.11022E-10	2.22044E-10
6:40	1048576	2097151	2.22045E-10	4.44089E-10
7:00	2097152	4194303	4.44089E-10	8.88178E-10
7:20	4194304	8388607	8.88178E-10	1.77636E-09
7:40	8388608	16777215	1.77636E-09	3.55271E-09
8:00	16777216	33554431	3.55271E-09	7.10543E-09
8:20	33554432	67108863	7.10543E-09	1.42109E-08
8:40	67108864	134217727	1.42109E-08	2.84217E-08
9:00	134217728	268435455	2.84217E-08	5.68434E-08
9:20	268435456	536870911	5.68434E-08	1.13687E-07
9:40	536870912	1073741823	1.13687E-07	2.27374E-07
10:00	1073741824	2147483647	2.27374E-07	4.54747E-07
10:20	2147483648	4294967295	4.54747E-07	9.09495E-07
10:40	4294967296	8589934591	9.09495E-07	1.81899E-06
11:00	8589934592	17179869183	1.81899E-06	3.63798E-06
11:20	17179869184	34359738367	3.63798E-06	7.27596E-06
11:40	34359738368	68719476735	7.27596E-06	1.45519E-05
12:00	68719476736	1.37439E+11	1.45519E-05	2.91038E-05
12:20	1.37439E+11	2.74878E+11	2.91038E-05	5.82077E-05
12:40	2.74878E+11	5.49756E+11	5.82077E-05	0.000116415
1:00	5.49756E+11	1.09951E+12	0.000116415	0.000232831
1:20	1.09951E+12	2.19902E+12	0.000232831	0.000465661
1:40	2.19902E+12	4.39805E+12	0.000465661	0.000931323
2:00	4.39805E+12	8.79609E+12	0.000931323	0.001862645
2:20	8.79609E+12	1.75922E+13	0.001862645	0.00372529
2:40	1.75922E+13	3.51844E+13	0.00372529	0.007450581
3:00	3.51844E+13	7.03687E+13	0.007450581	0.014901161
3:20	7.03687E+13	1.40737E+14	0.014901161	0.029802322
3:40	1.40737E+14	2.81475E+14	0.029802322	0.059604645
4:00	2.81475E+14	5.6295E+14	0.059604645	0.11920929
4:20	5.6295E+14	1.1259E+15	0.11920929	0.238418579
4:40	1.1259E+15	2.2518E+15	0.238418579	0.476837158
5:00	2.2518E+15	4.5036E+15	0.476837158	0.953674316
5:20	4.5036E+15	9.0072E+15	0.953674316	1.907348633
5:40	9.0072E+15	1.80144E+16	1.907348633	3.814697266

Should include the formulas used to generate this data. Did this student design this spreadsheet?

Apprentice

	6:00	1.80144E+16	3.60288E+16	3.814697266	7.629394531
	6:20	3.60288E+16	7.20576E+16	7.629394531	15.25878906
	6:40	7.20576E+16	1.44115E+17	15.25878906	30.51757813
	7:00	1.44115E+17	2.8823E+17	30.51757813	61.03515625
*	7:20	2.8823E+17	5.76461E+17	61.03515625	122.0703125
	7:40	5.76461E+17	1.15292E+18	122.0703125	244.140625
	8:00	1.15292E+18	2.30584E+18	244.140625	488.28125
	8:20	2.30584E+18	4.61169E+18	488.28125	976.5625
	8:40	4.61169E+18	9.22337E+18	976.5625	1953.125
	9:00	9.22337E+18	1.84467E+19	1953.125	3906.25
	9:20	1.84467E+19	3.68935E+19	3906.25	7812.5
	9:40	3.68935E+19	7.3787E+19	7812.5	15625
	10:00	7.3787E+19	1.47574E+20	15625	31250
	10:20	1.47574E+20	2.95148E+20	31250	62500
	10:40	2.95148E+20	5.90296E+20	62500	125000
	11:00	5.90296E+20	1.18059E+21	125000	250000
	11:20	1.18059E+21	2.36118E+21	250000	500000
	11:40	2.36118E+21	4.72237E+21	500000	1000000
	12:00	4.72237E+21	9.44473E+21	1000000	2000000
			2000000	2.11758E-16	
			pounds	pounds per bacteria	

Apprentice



Practitioner

Today we were presented with the math portfolio problem called Baffling Bacteria. In this task, I am asked to find out at what time of day would the colony of bacteria out weigh me. I am presented with the information that a bacteria reproduces every twenty minutes. If one bacteria enters my mouth at noon today, by tomorrow noon their would be 2,000,000 pounds of strep throat bacteria in my throat.

① The first thing I did was to think about this problem. I would need to record the data that I collected so I decided to use a table.

Please See Attached Table

②Now, I knew that the bacteria reproduced three times every hour.

Practitioner

So, I needed to figure out how many times the bacteria reproduced in one day. I found that by multiplying 24 times three I came to the conclusion that the bacteria reproduces 72 times in the course of one day.

③ I figured out that I needed to find out how much each bacteria weighed. I used a table and I kept track of the reproduction in the bacteria. Then I tried to divided 2,000,000 by two 72 times. The caculator I was using did not have the capacity to do this, so I decided to try half of 72 which is 36. So I did that, I took the number 2,000,000 and divided it by two 36 times. It worked. I got the answer 0.0000241. So the bacteria would weigh 0.0000241 at midnight.

Please See Attached Table

Used good reasoning here in dealing with a non-scientific calculator.

Practitioner

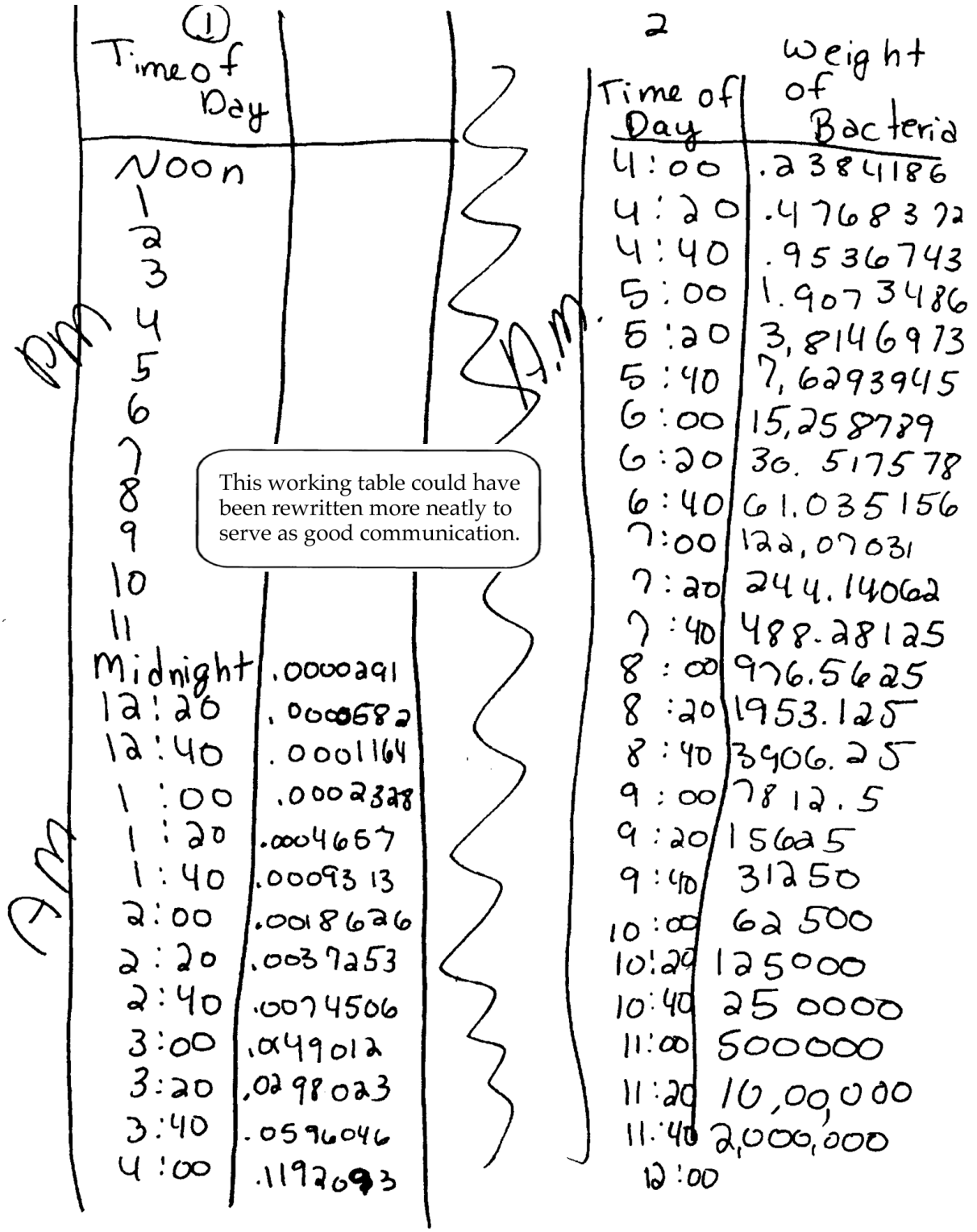
(1) After that I kept filling in the weight by multiplying times two until I got to my body weight. It was the bacteria weighed 61,035,156 pounds and at 7:00 weighed 122.07031 pounds. I weigh night in the middle of that so at approximately 6:50 the bacteria would outweigh me.

Now, in real life, that wouldn't happen. First of all, the bacteria would not go unnoticed for that period of time. Also, your throat does not have the capacity, or size, to hold all of that bacteria. This was also like one of my other problems, Miracle Mile, where the Rookies pay went up by multiplying by two's, otherwise called expunential growth. This chart uses exponential growth also.

Good estimation powers used here. More good mathematical reasoning.

Shows recognition of underlying math concept of the task and compares to another task, but could develop that further.

Practitioner



(1)

Time of Day	
Noon	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Midnight	.0000291
12:20	.0000582
12:40	.0001164
1:00	.0002328
1:20	.0004657
1:40	.0009313
2:00	.0018626
2:20	.0037253
2:40	.0074506
3:00	.014901 2
3:20	.0298023
3:40	.0596046
4:00	.1192093

This working table could have been rewritten more neatly to serve as good communication.

(2)

Time of Day	Weight of Bacteria
4:00	.2384186
4:20	.4768372
4:40	.9536743
5:00	1.907 3486
5:20	3,8146973
5:40	7,6293945
6:00	15,258789
6:20	30. 517578
6:40	61.035156
7:00	122,07031
7:20	244.14062
7:40	488.28125
8:00	976.5625
8:20	1953.125
8:40	3906.25
9:00	7812.5
9:20	15625
9:40	31250
10:00	62 500
10:20	125000
10:40	25 0000
11:00	500000
11:20	10,00,000
11:40	2,000,000
12:00	

Expert

HEAVYWEIGHT BACTERIA

My weight is about 61 pounds. The bacteria weigh 2,000,000 pounds after 24 hours, having doubled in weight ever 20 minutes.

2000000 / 61 ≈ 32787 (rounded to the nearest integer)

2 15th = 32768 which is very close to 32787

So the bacteria double in weight just over 15 times after they weighed the same as me.

15 X 20 mins = 300 mins = 5 hours

So the bacteria weight the same as me about 5 hours before they weight 2,000,000 pounds. The first bacterium was inhaled at 12 noon and 24 hours later would be 12 noon the next day. 5 hours earlier than that would be **7:00 a.m. the next day, when the bacteria weight about the same as me.**

As a check, assume the bacteria weighed exactly 61 pounds at 7:00 a.m. the next day, when the bacteria weighed about the same as me.

Good use of math language.

Time	Weight of Bacteria (in pounds)
7:00 a.m.	61
7:20 a.m.	122
7:40 a.m.	244
8:00 a.m.	488
8:20 a.m.	976
8:40 a.m.	1,952
9:00 a.m.	3,904
9:20 a.m.	7,808
9:40 a.m.	15,616
10:00 a.m.	31,232
10:20 a.m.	62,464
10:40 a.m.	124,928
11:00 a.m.	249,856
11:20 a.m.	499,712
11:40 a.m.	999,424
12 noon	1,998,848

Good method of verification.

Good mathematical comment on solution.

The chart shows that if the bacteria weighed exactly 61 pounds at 7:00 a.m., that they would weigh exactly 1,998,848 pounds at 12 noon (24 hours after start). 1,998,848 is 1,152 less than 2,000,000 which is an error of only 0.0576%.