The Twelve Days of Christmas

Most of us are familiar with the traditional Christmas carol, *The 12 Days of Christmas*. Although there are many variations about what is given on each particular day, let us use the following version:

On the 1st day of Christmas, my true love gave to me a partridge in a pear tree.

On the 2nd day of Christmas, my true love gave to me 2 turtledoves and a partridge in a pear tree.

On the 3rd day of Christmas, my true love gave to me 3 French hens, 2 turtledoves and a partridge in a pear tree.

(the song continues and the last verse is:)

On the 12th day of Christmas, my true love gave to me 12 drummers drumming, 11 pipers piping, 10 lords a-leaping, 9 ladies dancing, 8 maids a-milking, 7 swans a-swimming, 6 geese a-laying, 5 GOLDEN RINGS; 4 calling birds, 3 French hens, 2 turtledoves and a partridge in a pear tree.

- How many of each gift does the true love actually give?
- Which gift(s) are given in the greatest quantity (not necessarily most often)?
- Which gift(s) are given in the smallest quantity (not

necessarily least often)?

- Discuss any patterns you discover.
- Suppose instead there were 15 days of Christmas. Which day's gifts would be given in the greatest quantity? Why? How many?
- What if there were 20 days? 25 days?
- Discuss what you have learned in this project.

Grade Levels 6 - 8

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Context

This is a project that engages students around the holidays. Students who know the song are astounded that so many gifts are received. Most think that only 12 gifts are given.

This project has also been done with younger students.

What This Task Accomplishes

This problem provides students with a real challenge. They recognize how important it is to read problems closely. They can recognize patterns of numbers that emerge.

What the Student Will Do

Students will need to think through what the song says very carefully. They will need to set up a "system" to solve the problem. They will recognize patterns and symmetries.

Time Required for Task

1-2 hours

One period is a bit tight for finishing this problem. It should typically take two periods to give students the opportunity to think through the problem and set up and discuss solutions.

Interdisciplinary Links

Students can be asked to look for other mathematics problems in music. It can also be integrated with history and mythology (for example, the symbolic meaning of the presents). Students might want to discuss what other religions have holidays where gifts are given? What is the genesis of gift giving?

Teaching Tips

Students need to be encouraged to think carefully through the problem. Graphing responses will help students see patterns in the numbers.

Suggested Materials

- Paper
- Pencils
- Calculators

Possible Solutions

The Expert benchmark has a good discussion of the correct answers. There were 364 gifts given. There is symmetry, with the gifts given at the beginning and end of the period representing a smaller proportion of the total, while gifts given in the middle period represent a larger share of the total. If there is an odd number of days (15) there is one middle day when the most gifts are given. If there is an even number of days, there will be two days when the most gifts are given.

Benchmark Descriptors

Novice

Although the Novice arrives at the right answer to the first part of the question, s/he uses faulty

reasoning and has an incomplete strategy. There is no apparent strategy for an overall solution. S/he applies inappropriate concepts. S/he did not complete the problem. For the 15 days of Christmas s/he has the wrong answer.

Apprentice
The Apprentice has a partially useful strategy for arriving at an answer, but never provides an answer to the first part of the problem. S/he missed "seven swans" in that part of the problem as well. The communication is not clear in several areas, but especially in part four where s/he is asked to describe a pattern. In part five the answer is incorrect.

Practitioner
This response really lies between Apprentice and Practitioner. The answers are here, but the strategy is not clear. Lists, as in part 1, are not mathematical representation. The reasoning is not communicated completely.

Expert
There is sophisticated reasoning, insight into patterns. Communication is very clear and there is an excellent use of mathematical representation.