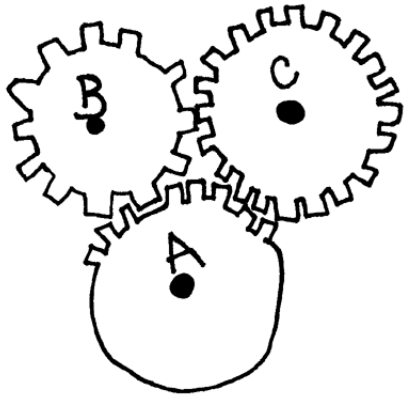
Gears

Look at the set of gears. Gear B has 12 teeth and gear C has 20 teeth. Find the number of teeth that Gear A has to have so that when it rotates once, Gear B and Gear C have rotated some whole number of times. You may use the gears we were experimenting with to help solve this problem.

Be sure to explain your reasoning clearly. Connect as much mathematics and use as much math language as you can.



Grade Levels 6 - 8

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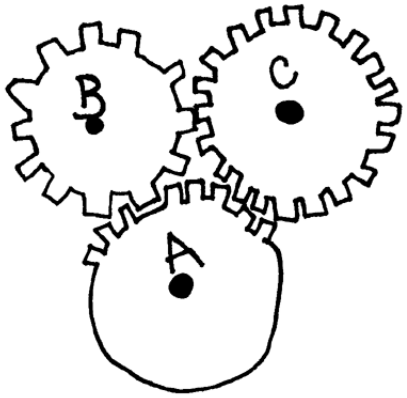


Context

My sixth grade class was just beginning to study gears in science class and had already studied multiples in math class. I wanted to integrate and connect the concepts that were being presented in both classes. The class had been playing with some gears we made out of cardboard, investigating what direction interlocking gears turned, and putting different size gears together to see how they turned. As they collected data on their gear ratios many were beginning to see a relationship.

What This Task Accomplishes

This task pulls together some of the investigations the students were conducting. I was hoping that most students would recognize the use of multiples in determining gear ratios. We had not studied ratios formally, but had some discussions about ratios and proportions in other problems. Some students may be able to connect those ideas also.

Standard 4 in the *NCTM Standards-Mathematical Connections*- states that, "In grades 5-8 the mathematics curriculum should include the investigation of mathematics connections so that

students can apply mathematical thinking and modeling to solve problems that arise in other disciplines such as art, music, psychology, science and business."

What the Student Will Do

Some students used the cardboard gears while others felt confident to imagine the gears turning. Most students used a chart to keep track of the turns the gears made.

Time Required for Task

45 minutes

Interdisciplinary Links

This task could be used with a science unit studying gears.

Teaching Tips

Having gears that students can handle would be helpful, especially if this will not be coordinated with a science unit. I had templates of gears and cut them out of cardboard and used straight pins as the axle.

Suggested Materials

Cardboard gears

Possible Solutions

The least common multiple of 12 and 20 is 60, so gear A has to have 60 teeth.

Benchmark Descriptors

Novice
There is no evidence of a strategy or an answer to the problem. The student shows two multiplication problems that each equal 160, but the numbers do not relate to the problem. There is no explanation of the solution. The student tries to place the gears differently, but that does not seem to help solve the problem.

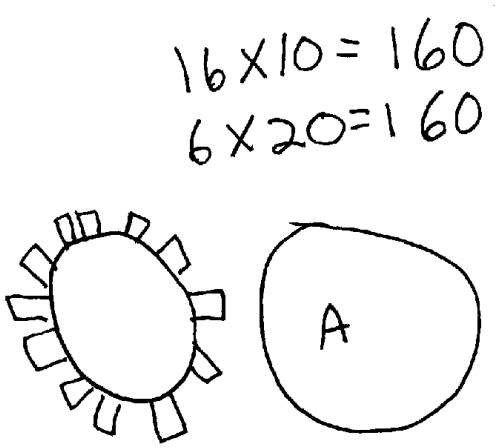
Apprentice
The student uses a strategy that is partially useful. Their use of multiples is evidence of mathematical reasoning, but they fail to find the least common multiple and therefore Gear A will turn more than one time. There is some use of mathematical representation and language.

Practitioner
This student shows a broad understanding of the problem. His/her use of the chart allowed a

pattern to be seen. The clear explanation shows that the student visualized the gears moving. The connection to multiples is made and used to find the solution. The student uses effective representation to collect data and find a pattern.

Expert
This student shows a deep understanding of the problem. S/he not only made the connection to multiples, but also talks about ratios and shows how the ratio was determined. The solution leads directly to a solution and the student employs refined reasoning when they simplified their findings to make ratios. There is also a clear and effective explanation using precise and appropriate mathematical language (multiples, LCM, ratio), and the chart helps communicate the pattern that leads to the solution.

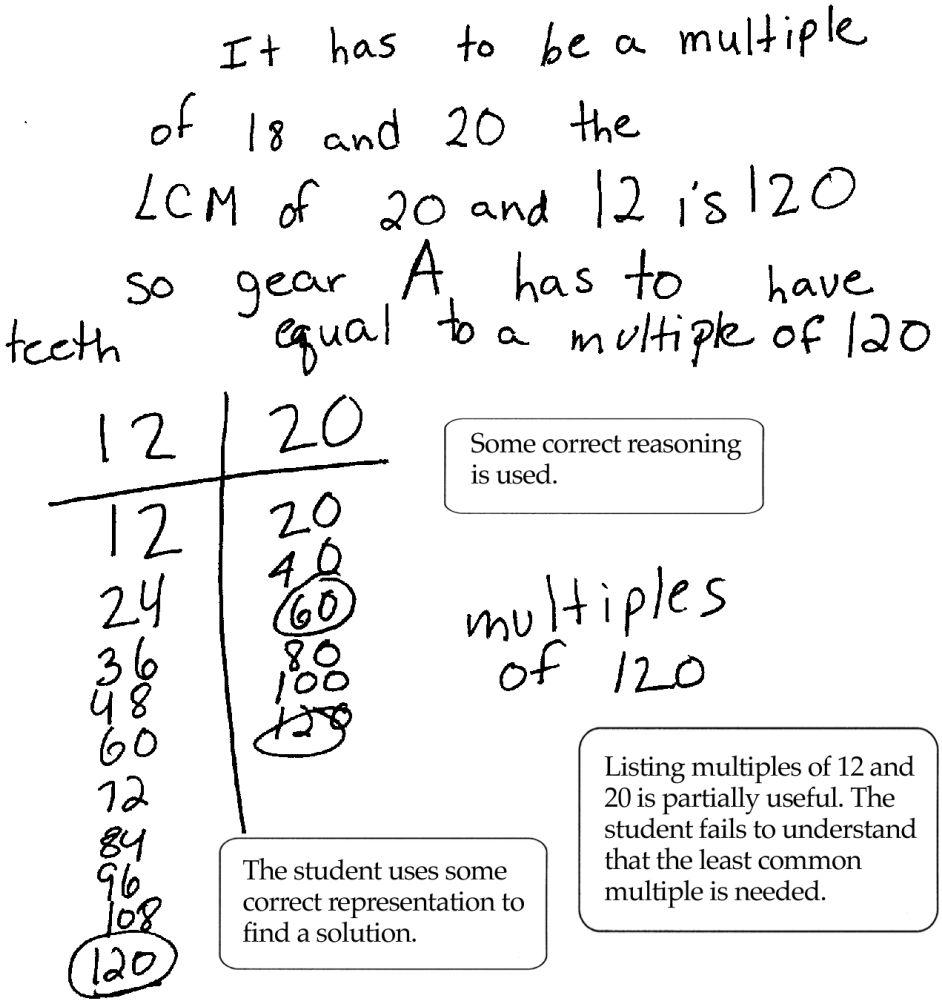
Novice



No correct reasoning is evident. These numbers have no correlation to the problem.

This diagram does not enhance the student's solution and acts more as a "decoration".

Apprentice



Practitioner

gear A	gear B	gear C	turns
60	12	20	1)
1	24	40	2)
	36	60	3)
	48	80	4)
	60	100	5)

This representation assists in communicating the solution.

The student's approach and reasoning is explained.

I got my answer by making a chart. Then I found out that if I roatated The gear one turn. Twelve would have doubled as would twenty because B and C would of turned all the way around. If I turned it again (my third turn) it would increase B by 12, making it 36 and C would of increase by

Practitioner

The student uses accurate and appropriate math language.

20 making it fourty. on turn four B increases by 12 to fourty-eight and C increases to again by twenty equalling to eighty. On turn five B increased by 12 equalling it to be 60 and C increased by twenty equalling one hundred. When I got to turn five I noticed that two multiples were the same — sixty. So that made consider that A had sixty teeth that was the first place that I saw they macked.

A correct solution is achieved.

Expert

You just have to figure out what the GCM is for 12 and 20

12 24 36 48 (60) LCM 60.
20

The student's efficient approach leads to a correct solution.

The ratio for B will be 1:5, the ratio C will be 1:3

B will go around 5 whole times and C will go around 3 whole times.

Both GEARS B and C have to rotate on even number of times to do this GEAR A has to have a number of teeth that is a multible of 12 and 20 The LCM between these numbers is 60, the number of

12 12, 24, 36, 48 (60) LCM 60
20 2

The student makes mathematically relevant observations.

12 x 5 = 60 (the LCM)
20 x 3 = 60 (the LCM)

$\frac{12}{60} = \frac{1}{5}$ $\frac{20}{60} = \frac{1}{3}$

These fractions in their simplist forms are 1:5 and 1:3. The rat is

The student uses precise math language to communicate.

	Multiply by 1	Multiply by 2	Multiply by 3	Multiply by 4	Multiply by 5
Multiples of 12	12	24	36	48	(60)
Multiples of 20	20	40	(60)		

60 is the least common multiple (LCM) between 12 and 20.

This is an accurate and well-labeled diagram.