Fences for Grazing

Create a fenced in space with the maximum area for your cow Bessie, given 100 feet of fencing.

How many poles would you have for this area if you need 1 pole every 5 feet?

How do you know it is the maximum area? Explain.

What is that area? Explain.

Now, instead of having the grazing area in the middle of a field you decide to use a side of your barn. With the same amount of fence and the side of the barn being 50 feet, find the maximum area of this alternative grazing pasture.

How do you know this is the maximum area? Explain

What is the area? Explain

Compare the 2 answers and reasoning behind them. What do they suggest? What conjectures can you make?

Extend your thoughts.

Grade Levels 6 - 8

Fences for Grazing

Create a fenced in space with the maximum area for your cow Bessie, given 100 feet of fencing.

How many poles would you have for this area if you need 1 pole every 5 feet?

How do you know it is the maximum area? Explain.

What is that area? Explain.

Now, instead of having the grazing area in the middle of a field you decide to use a side of your barn. With the same amount of fence and the side of the barn being 50 feet, find the maximum area of this alternative grazing pasture.

How do you know this is the maximum area? Explain

What is the area? Explain

Compare the 2 answers and reasoning behind them. What do they suggest? What conjectures can you make?

Extend your thoughts.

Context

We had been studying area and perimeter. I wanted a problem that would use the two concepts, and hopefully students would come to some conclusions about squares and maximizing area.

What This Task Accomplishes

This task will tell me which students are comfortable with working with area. It will tell me which students have grasped the concept of area and perimeter and can work with and express these concepts.

What the Student Will Do

Most students started by drawing a diagram. Some played around with a number of possible rectangles as they tried to get the largest area.

Time Required for Task

50 minutes

Interdisciplinary Links

This type of problem can be used with any planning activity students might be doing, designing playgrounds, for example.

Teaching Tips

Encourage students to test other rectangles. Question students as to whether they could find a rectangle with a larger area.

Suggested Materials

- Graph paper
- Calculator

Possible Solutions

Maximum area is actually a circle (though no one in my class thought of that) and using Pi as 3.14 the area is about 796.18 square feet. The more sides the polygon has the bigger the area will be with a given perimeter. The rectangle with the largest area is 25' x 25' with an area of 625 sq ft. A square maximizes the area of rectangles with a given perimeter. You will need 20 fence posts five feet apart.

The rectangle with the largest area using the side of the barn is 25' x 50' with an area of 1,250 sq ft. But, again, if a student used polygons with increasing sides, drew a diagram to scale and accurately approximated the area, that would be an indication of a deep understanding of the problem. The maximum area using the side of the barn would be to use the whole side of the barn and curve the 100' of rope into half an oval. Some students may draw the diagram to scale and approximate the area.

Benchmark Descriptors

Novice
This student is applying inappropriate concepts of area and perimeter. His/her strategy does not help to solve the problem. There is little explanation and inaccurate use of a diagram.

Apprentice
The student uses a strategy that is partially useful. S/he uses all the fencing in the second part, but not the first part. There is some evidence of reasoning, but they could not completely carry out the mathematical procedures. The explanation is incomplete. There is some use of mathematical representation.

Practitioner
This student shows a broad understanding of the problem. They use a strategy that will lead to a solution of the problem for rectangles. They use effective mathematical reasoning with a clear explanation and appropriate use of mathematical representation and terminology.

Expert
The Expert has a deep understanding of the problem and identified the appropriate mathematical concept (maximizing the area by keeping the three sides equal). S/he uses a sophisticated strategy (use of Pythagorean Theorem) that uses refined and complex reasoning. There is an effective explanation (using mathematical equations to communicate). Mathematical representation is used to help communicate ideas.

Novice

The student makes a rudimentary observation, which lacks evidence on how this fact was determined.

1. How do you know it is the maximum area? Explain.
   Im not sure
2. What is that area? Explain.
   I got the area 100 because I counted 10 and 10 across and got 100.

Now, instead of having the grazing area in the middle of a field you decide to use a side of your barn. With the same amount of fence and the side of the barn being 50 feet, find the maximum area of this alternative grazing pasture.

The student makes a rudimentary observation, which lacks evidence on how this fact

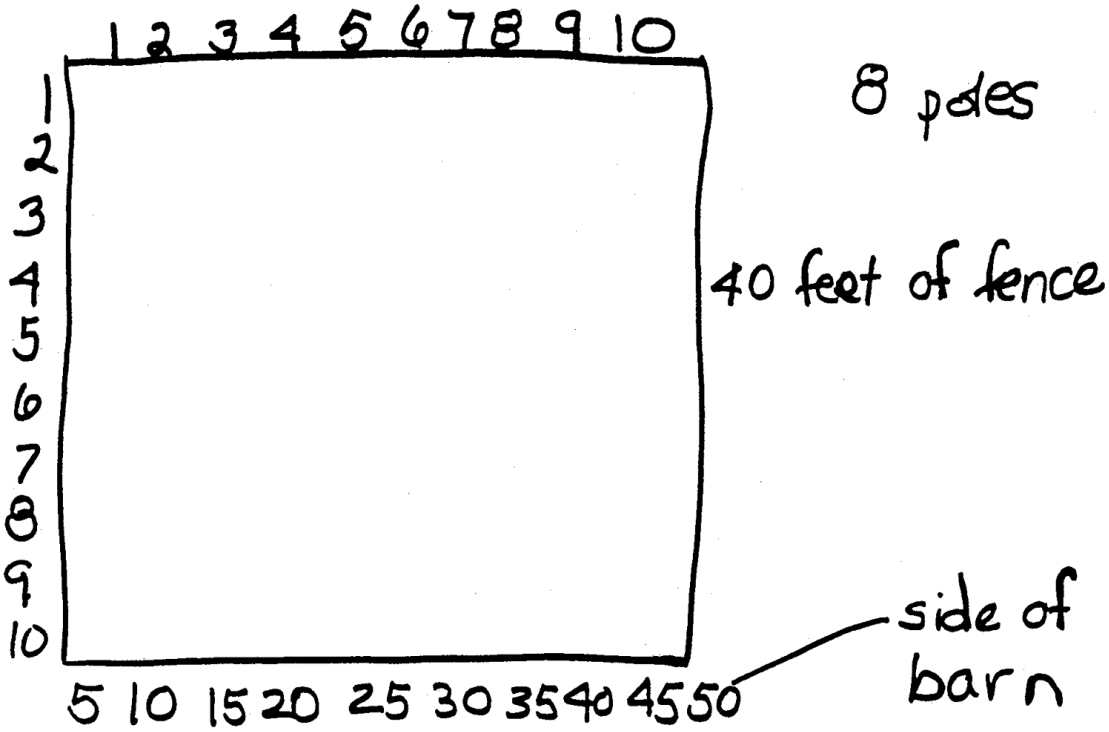
1. How do you know this is the maximum area? Explain. I used all the 250 the fence and the barn side. Because

2. What is the area? Explain.

The solution lacks correct reasoning.

3. Compare the two answers and reasoning behind them. What do they suggest? What conjectures can you make? Extend your thoughts.
   the more fence you have the more pasture you can have.

Novice



The student creates a diagram to communicate his/her incorrect solution.

No part of the solution is correct.

25
x10
1250= area

Apprentice

The solution lacks reasoning.

1. How do you know it is the maximum area? Explain.
I'm not sure. I gussed.
2. What is that area? Explain.
well I got the area 100 because I counted down 10 and 10 across and got 100.

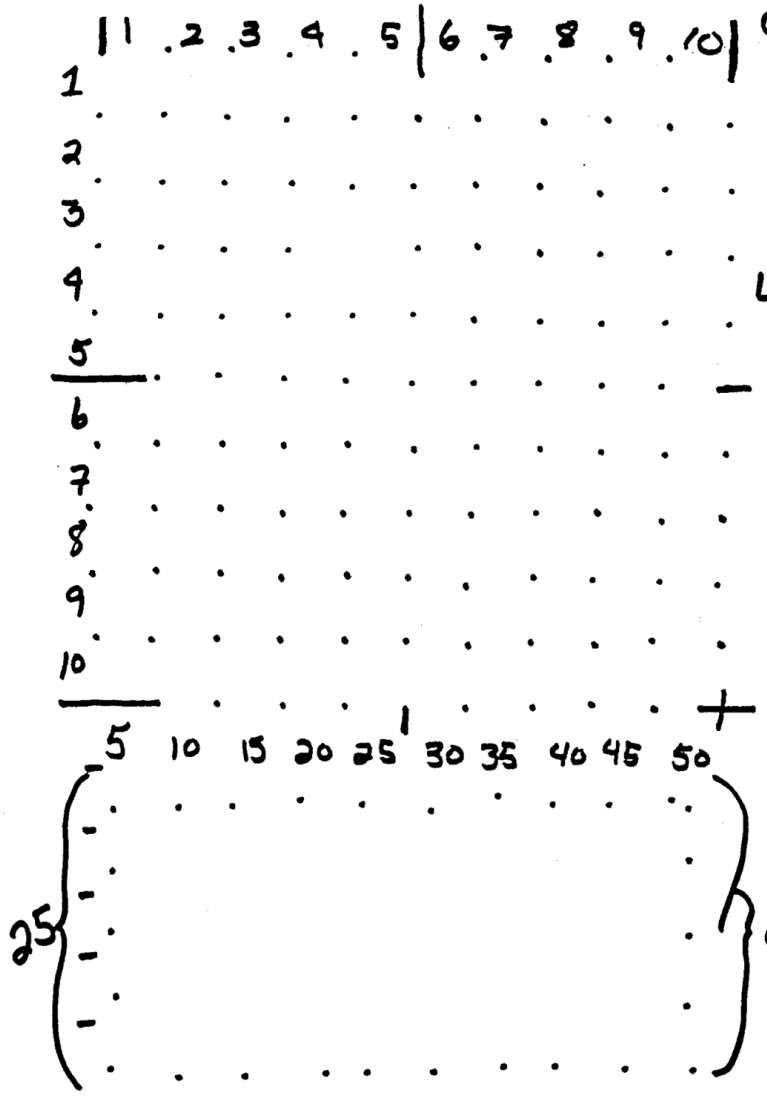
Now, instead of having the grazing area in the middle of a field you decide to use a side of your barn. With the same amount of fence and the side of the barn being 50 feet, find the maximum area of this alternative grazing pasture.

1. How do you know this is the maximum area? Explain. Because I used all the fence and the barn side.
2. What is the area? Explain. The area is 250 because I timesed 25 x 50

3. Compare the two answers and reasoning behind them. What do they suggest? What conjectures can you make? Extend your thoughts.
the more fence you have the more pasture you can have.

This is a rudimentary observation.

Apprentice



The student uses diagrams to communicate his/her incorrect solution.

The student neglects to use all 100 feet of fencing.

All 100 feet of fencing is used for this part of the problem.

The student uses basic math language to communicate.

Practitioner

The student makes mathematically relevant observations.

First I found all the rectangles with the perimeter of 100 What I came up with was 1 x 49, 2 x 48, 3 x 47 4 x 46, 5 x 45, 6 x 44, 7 x 43.

When I got to here I noticed a pattern that the dimension all added up to 50 So there would be 25 possibilitys because when you get to 25 x 25 and start going the way you will have already done it just before.

ex. 24 x 26, 25 x 25, 26 x 24

same

I then I notice how that 25 x 25 must be the biggest area because it's a square and will have the least amount of edges showing on each square.

Practitioner

A2 To find the perimeter all you have to do is multiply both the dimensions together so 25x25=625 which is the area.

The student successfully addresses the first aspect of the problem.

A3 To find how many poles I need I just divided 100÷5=20 because I needed a pole every 5 feet and I had 100 feet. I needed 20 poles.

This part of the task is successfully addressed as well.

B1 Now for this I knew that 2 dimensions had to be 50 feet to make a rectangle so what I did was subtract that 100 feet from my 50 feet total perimeter. I had 50 feet left so I divided it in half and got 25 so my rectangle would be 50x25. I know this is the largest area because

The student communicates the solution in a clear logical order.

The student limits the solution possibilities by assuming the grazing area must be a rectangle.

Practitioner

it's the closest to a square rectangle I can make which means less amount of edges showing on each indivitual square.

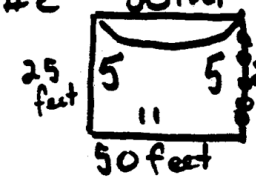
B2 To find the area all I have to do is 25 x 50 = 1250 which is the area.

The student continues to address all aspects of the problem.

B3 Answer #1 = 625 = 25x25
Answer # 2. 1250 = 25x50

One observation I made was that for #1 I needed 20 poles and for # 2 I only need 19 poles.

#2



no posts - barn

21 but I counted each Corner twice so I have to subtract 20-2=19

Also the 2# answer is double the 1# answer in area. Probably because

This is an interesting observation.

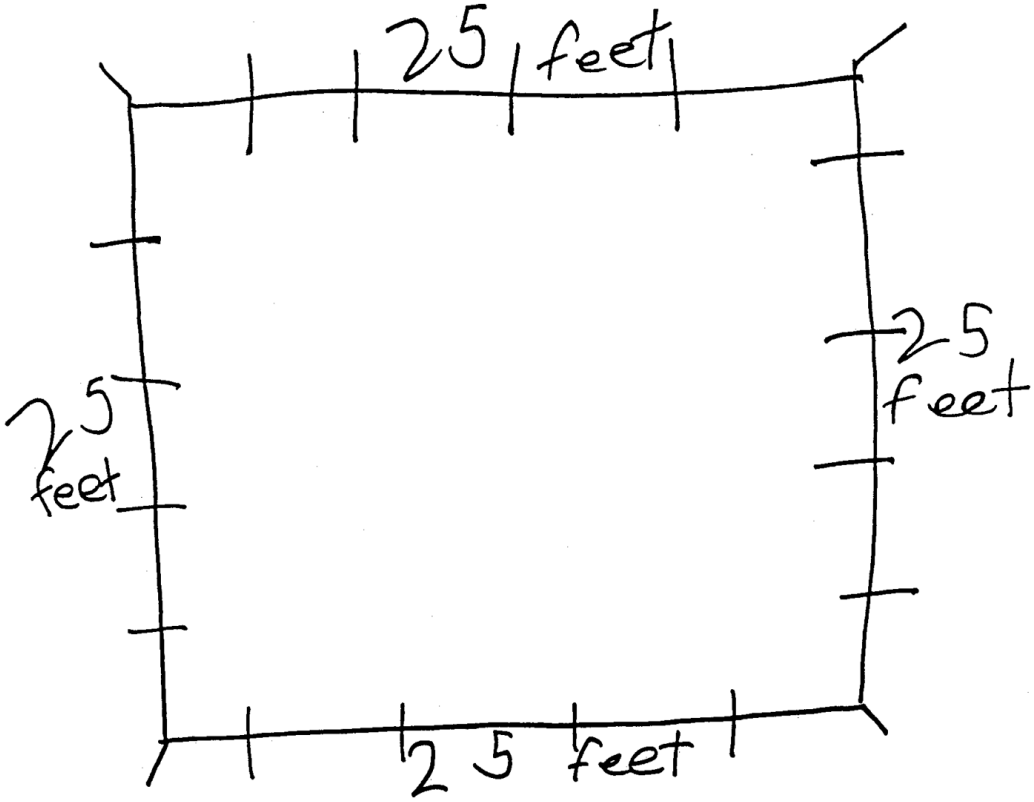
Practitioner

You are doubling one dimension which is really doubling the multiplication

$25 \times 25 = 625$
↓ doubled
$25 \times 50 = 1250$

If the barn side was 98 the only rec. you could make with 100 feet of fence would be 1×98 which only has the area of 98 so good thing my barn was only 50 feet.

Expert



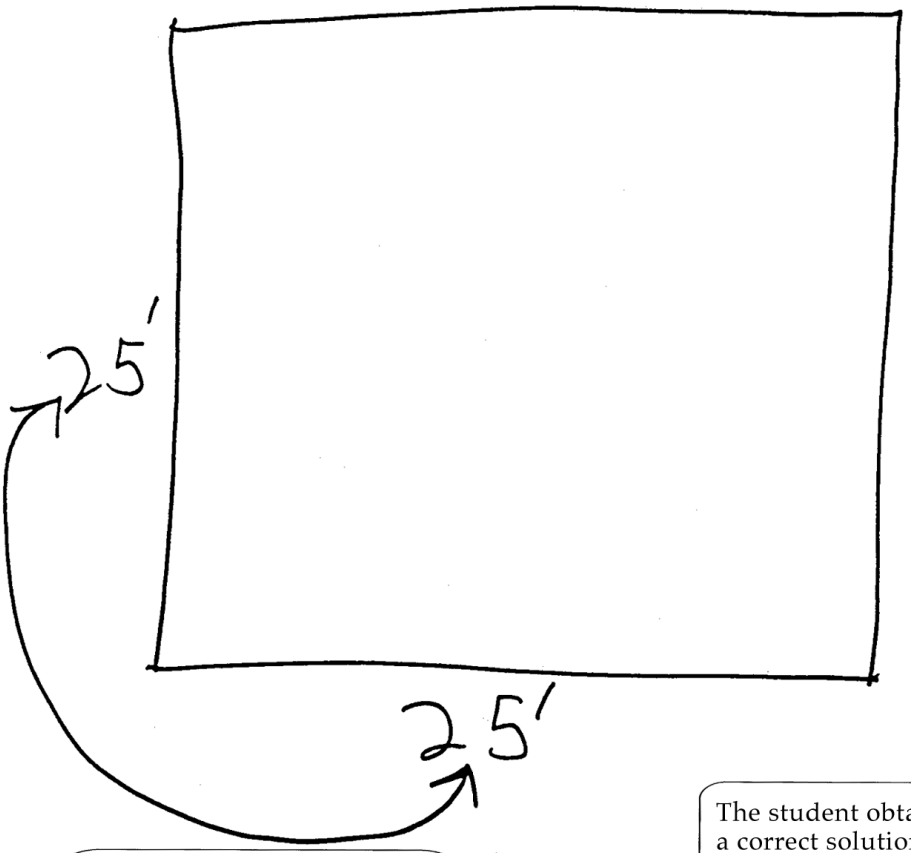
The student addresses the number of posts correctly.

The student creates an accurate and appropriate diagram.

5 goes into twenty-five five times so that means that there would be 5 posts on each side. So for the total amount of posts there would be 20 of them

4 sides 5 on each

Expert



The student obtains a correct solution.

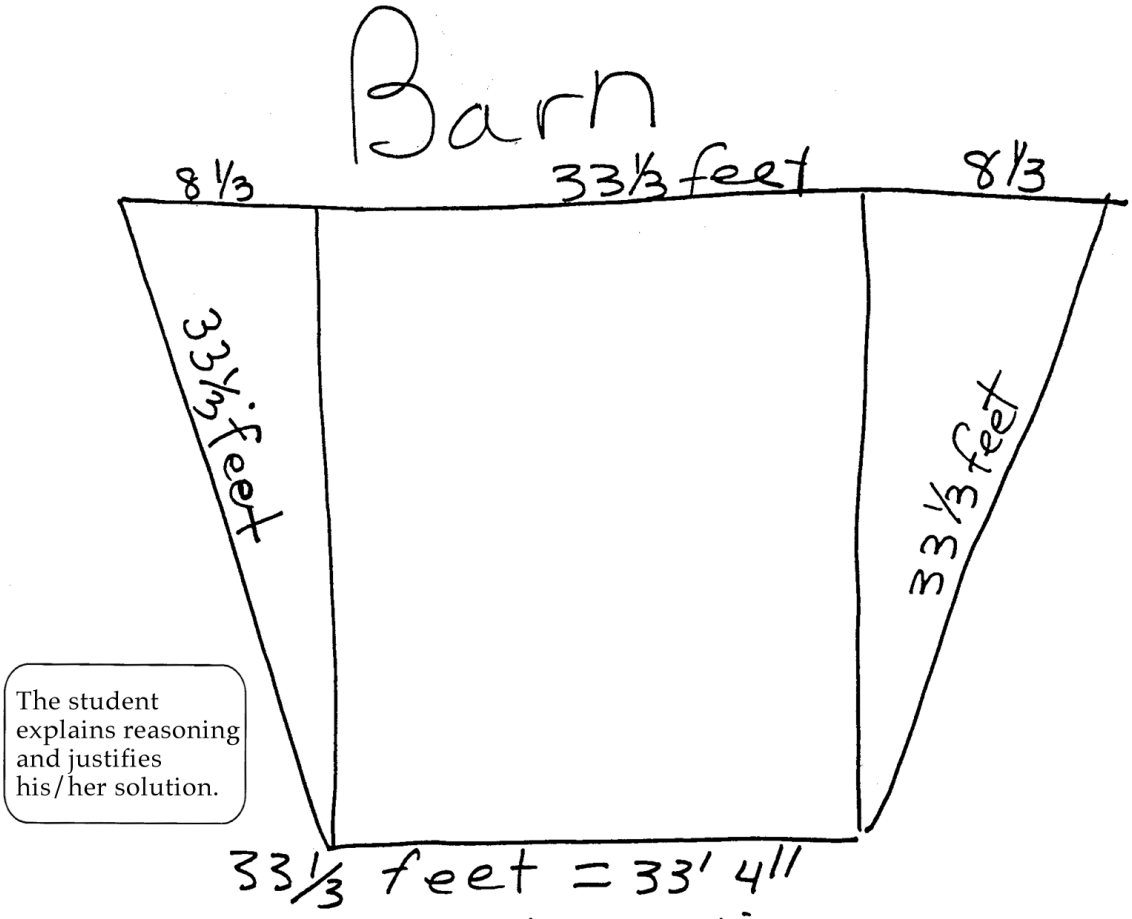
The student uses accurate and appropriate math language.

the area is 625sqft To get the area I multiplied 25 by 25.

I know this is the maximum area because I found out the square A square gives the biggest area if You use the same perimeter.

The student applies the correct mathematical reasoning.

Expert



The student explains reasoning and justifies his/her solution.

The matimum area of this grazing area is 1341.6 Square feet.

I know this is the maximum area because I multiplied 32.25 by 33 1/3 to the area of the rectangle Then I multiplied 8.3 x 32.25 to get the area of the 2 triangles. The total area is 1073.925 + 267.675 = 1341.6 sqft.

Expert

The difference between the Two grazing areas is 7166 square ~~feet~~ the grazing area that was made with the 50 foot side of the barn has more because it had the 50 foot side and had to have the other three sides. together equal 100 the other grazing area has to have the ~~four~~ sides equal 100 so it did not have a side with more feet.

So I think that Bessie would rather have the grazing area that was made with the barn so she has more grass to eat and more room to walk around.

The student addresses all the aspects of this multi-step problem.