Runners' Dilemma

There are 2 friends chasing one another. They are 80 feet apart. Caroline is chasing after Sam. Sam runs at a speed of 22 feet per second and Caroline runs at a speed of 31 feet per second. How many seconds will it take for Caroline to catch Sam?

Think of a way to represent your findings, but also fully explain your thinking.

Grade Levels 6 - 8

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Context

My sixth grade class just finished reviewing long division, and they needed something else to remind them that there was more to mathematics than long division. When I think of a problem, I usually think of how my sixth graders would solve it. They are really into making charts for organizing data, so I figured they would use a chart to keep track of where Caroline and Sam were after running each second. I was completely blown away by some strategies that my students used to solve this problem. I have a self-assessment sheet that my students use to assess their problem-solving abilities. I have found this most valuable in communicating to my students what I value in a solution. One of the criteria is to try to make a formula that could generalize the problem. This has been one of the most powerful tools in getting students to extend their thinking. I had not thought of this problem as a problem easily generalized, but my students did. I was absolutely thrilled when they worked really hard to come up with a generalization and as they did, they also discovered a very efficient way of solving the problem that I had not thought about.

What This Task Accomplishes

This task has students generating and keeping track of data. Those students that solved the problem efficiently by dividing had to again deal with interpreting the remainder (many students still want to make the remainder a decimal in problem-solving situations - we have not divided decimals in class yet - and the division was so simple they did not need a calculator). This task lends itself to a generalization.

What the Student Will Do

Many students started by making a chart. I was very interested in the different strategies though. Some students started with a chart with Caroline at zero and Sam at 80 and added 31 and 22 respectively. Others realized that they could make a chart of Caroline and Sam at zero and add 31 and 22 respectively until Caroline and Sam's difference was 80. A few found that Caroline ran nine feet further each second and found how many nines were in the 80 feet.

Time Required for Task

45 minutes

Interdisciplinary Links

This problem could be nicely tied to a discussion of athletic skill or the differences between boys and girls.

Teaching Tips

I encouraged my students to come up with another way to represent their findings besides using a chart. I thought they knew enough graphing to come up with a good representation, however, I was very disappointed. If kids have had a formal study of graphing, this would be a good problem to make a double-line graph and see where Caroline and Sam's lines crossed.

Suggested Materials

- Graph paper
- Rulers

Possible Solutions

Depending on how exact the student chose to get, the possible solutions are eight seconds, 8/9 seconds, between eight and nine seconds or nine seconds.

One generalization could look like:

80/(C-S) = seconds, where C = Caroline's speed and S = Sam's speed.

Benchmark Descriptors

Novice
This student was hindered by the fact that they could not get a scale small enough to represent the distances and could not solve the problem. S/he also is trying to divide Caroline's speed by two, which will not help solve the problem.

Apprentice
The solution is not correct. Because of the incomplete explanation, I am not sure why the student decided on 11 seconds. S/he uses a strategy that is partially useful, but somehow the student does not see that in nine seconds Caroline is one foot ahead of Sam. Since s/he used two representations and came up with the same result indicates that part of the problem was not understood.

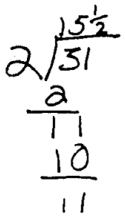
Practitioner
This student has a broad understanding of the problem. Making a chart of Caroline and Sam's

distance after each second is a useful strategy that leads to a solution (the minor calculation error in Sam's final distance being 279 instead of 278 is not significant). There is a clear explanation using two different types of representation (chart and diagram) and good use of mathematical terminology.

Expert
This student shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and information necessary for an efficient and sophisticated strategy. S/he employs refined and complex reasoning as s/he formulates the generalization. There is a clear and effective explanation. Mathematical representation is used to communicate the solution and there is precise and appropriate use of mathematical terminology and notation.

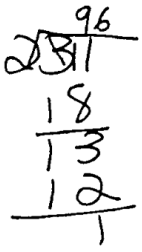
Novice

Dividing Caroline's speed by two will not help to solve the problem.



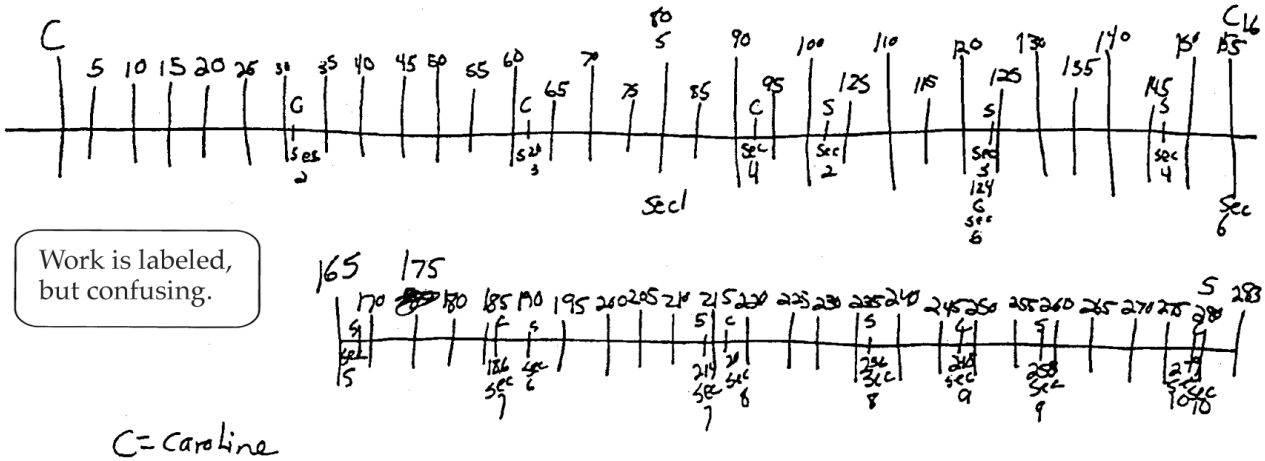
This diagram is not helpful as the student struggles to make it to scale.

Work lacks labels.



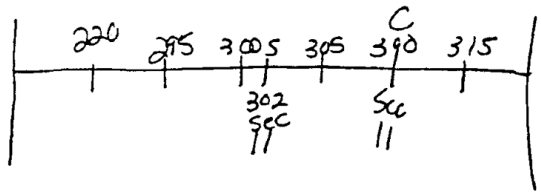
MAKE TO SCALE

Apprentice



Work is labeled, but confusing.

C=Caroline
S=Sam
tt=5feet
Sec=Second



it with take Caroline 11 seconds to catch Sam. every Second Caroline gains 9 steps on Sam because Sam runs 22 steps per sec. and Caroline runs 31 steps every sec. (The next way will be neater)

It is unclear how the student does not see that in second 9 Caroline is ahead of Sam.

It is unclear how the student obtains 11 seconds.

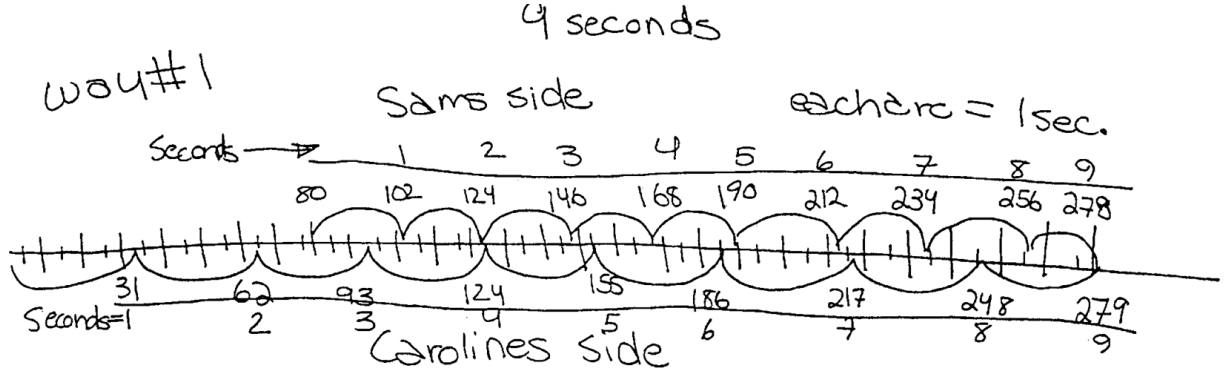
Second

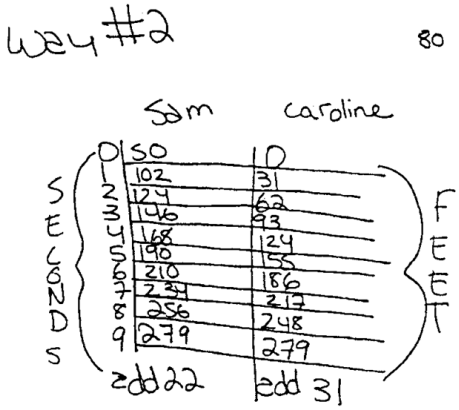
	1	2	3	4	5	6	7	8	9	10	11
Caroline	31'	62'	93'	124'	155'	186'	217'	248'	279'	310'	341'
Sam (starts 60 feet Ahead)	~~104~~	~~124~~	~~146~~	~~168~~	~~190~~	212'	234'	256'	278'	300'	322'

(Sam starts 8 feet further ~~Ahead~~)

This should be 80, not 8.

Practitioner





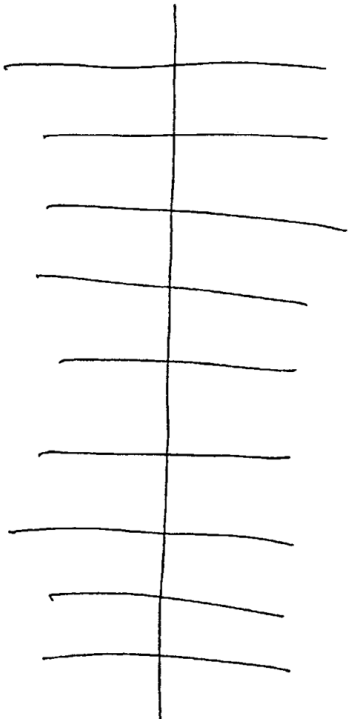
Caroline would catch Sam after the x had each one 279 feet.

*turnover

This chart is accurate and well labeled.

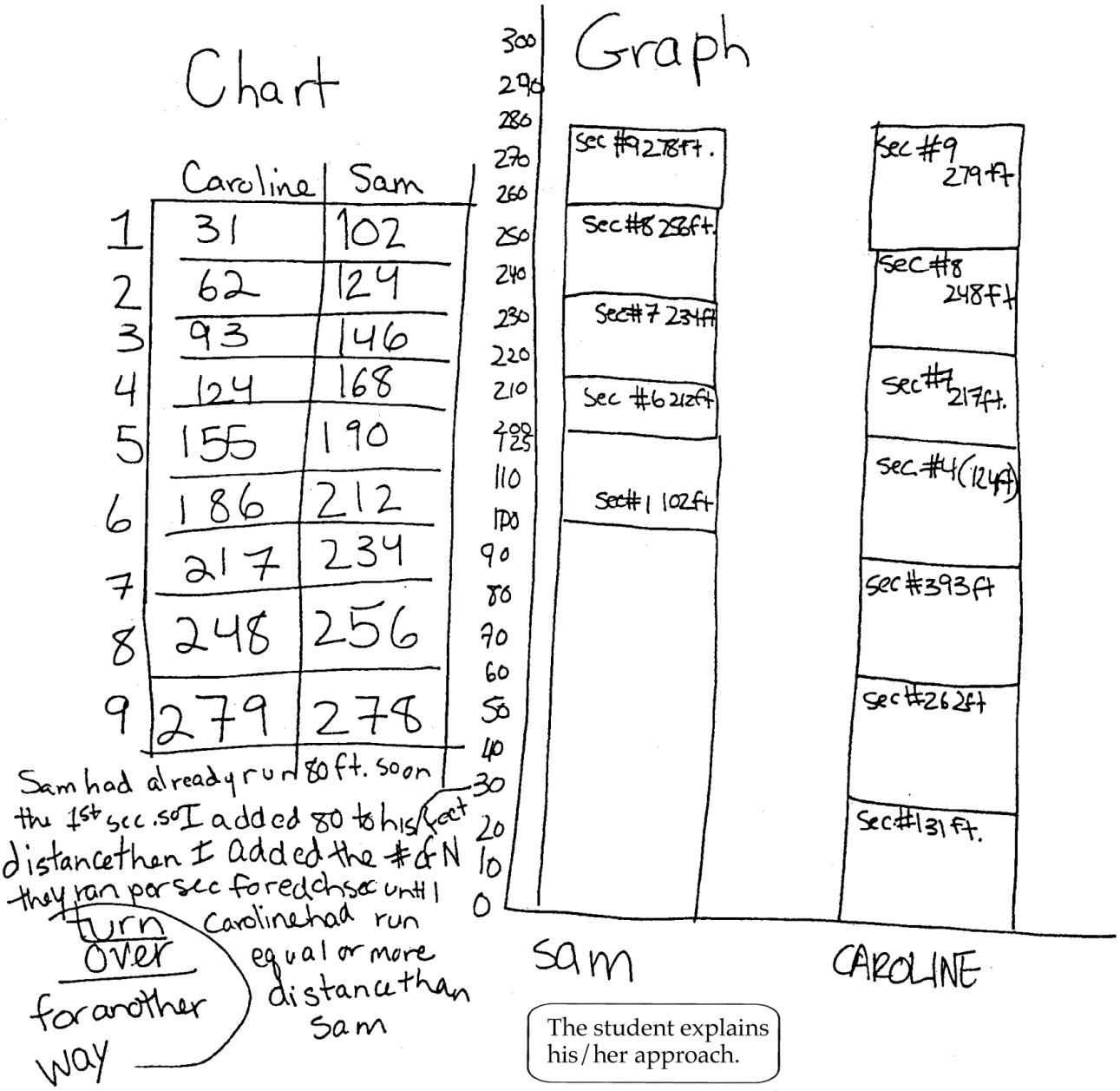
The student makes mathematically relevant observations.

The student clearly labels work.



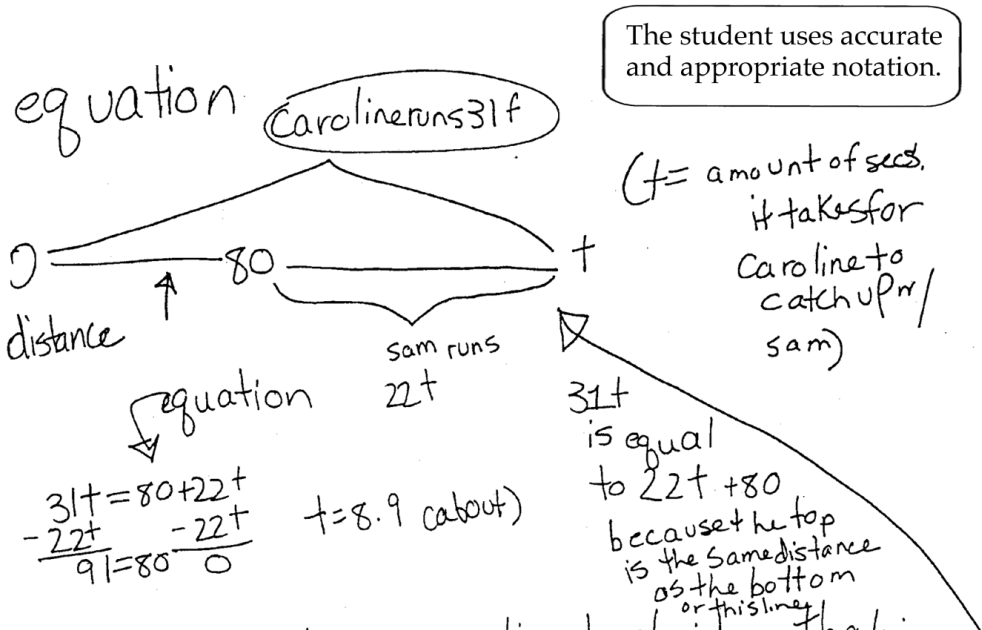
What I did was take each second and add on the # of steps she could go. each new set of steps I added on I sewed by the end they both had gone 278 feet. After I counted up all of the arcs that I made and counted each one side second. I don't think that was reasonable because you probally couldn't keep up the same pace through the whole thing I also noticed that the Caroline outran Sam by one foot so it was probally more like 8.8 or 9 seconds.

Expert



The student explains his/her approach.

Expert



I figured out an equation by deciding that in order to get even with Sam Caroline needed to run a certain amount of time (t) at a rate of 31 ft. per sec. Sam needed to run a certain amount of time (in order for caroline to catch up) at 22ft per sec but he had an 80ft. head start. So I came up with this equation: $31t = 80 + 22t$. Since I wanted to figure out what t was I took away things from the equation. I decided that t on one side and a # on the other was what I needed so I took away 22t from both sides and got: $9t = 80$ then I divided 80 by 9 which equals $8\frac{8}{9}$ secs. So $t = 8\frac{8}{9}$ secs. You can almost always include an equation in problems like this

The student uses accurate and appropriate notation.

The student is able to generalize the solution.

The student obtains a precise answer.