

Distribution Dilemma

Steve was hired to give out free movie tickets to customers during the grand opening of the new pizza parlor. On Friday, he gave out $\frac{1}{2}$ of his supply; on Saturday, he gave out $\frac{1}{3}$ of what was left; on Sunday, he gave out $\frac{1}{4}$ of the remaining amount; on Monday, he distributed $\frac{1}{3}$ of what was left; on Tuesday, he gave away $\frac{1}{6}$ of what remained and had 60 tickets left.

How many tickets did he have when he began to give them away?

Grade Levels 6 - 8

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Context

I was looking for a task that would encourage students to improve their mathematical representations. We have worked on visual models for problem solving this year and I hoped they would use such modeling in solving this task.

What This Task Accomplishes

This task requires the recognition of fractions and using common denominators to add and subtract fractions. It also provides the opportunity to use a visual model for problem solving. Students will encounter many problems of this type and having a workable strategy for this type of problem will serve them well.

What the Student Will Do

I had the students work alone on this task and encouraged them to work on their mathematical representations. Some students drew rectangles and divided them to represent each day's distributions. Other students worked backwards, starting with the 60 tickets that remained.

Time Required for Task

45 minutes

Interdisciplinary Links

There are no obvious interdisciplinary content links for this task. This problem-solving technique could be used to solve tasks written to complement any content area - historical topics, science units or literature connections.

Teaching Tips

We had studied area models of fraction representation before attempting this task. I had graph

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paper available to students. They are better able to subdivide rectangles using graph paper than using blank paper. They use a grid square to represent one unit more easily and accurately than estimating one unit on blank paper.

Being familiar with working backwards as a strategy will also help students experience success with this task.

Suggested Materials

- Grid paper
- Calculators
- Counting pieces of any sort (for acting out the problem)

Possible Solutions

Steve distributed 432 tickets in all. See Expert benchmark for details of solution.

Benchmark Descriptors

Novice

This student had the right idea with the rectangular area model. The fractional subdivision is accurate. The student fails to remember the fact that the final eight units represent 60 tickets rather than eight tickets. Since there is no attempt to verify the solution, the error is not picked up and the solution is incorrect. Willingness to invest more time in this activity would have undoubtedly rectified the error.

Apprentice

This student understood what was to be done to solve the problem. There appears to be no particular reason for the choice of a "12 x 12 box" for the model. When 60 tickets remain in a space containing 20 unit squares, the student fails to recognize the connection to three tickets/square and instead tries to subdivide the rectangle into 20 square subdivisions. This strategy unfortunately falls apart as the student attempts to deal with the extra 11 squares. Had the student checked the work with computation, the error would have been discovered.

Practitioner

This student understands the task, uses good mathematical representations and good mathematical language. The solution is accurate and well documented with diagrams. Solid mathematical reasoning was used throughout and a connection to past mathematical experiences is mentioned - though not expanded upon enough. Providing some form of verification of the solution would move this response to the Expert level.

Expert

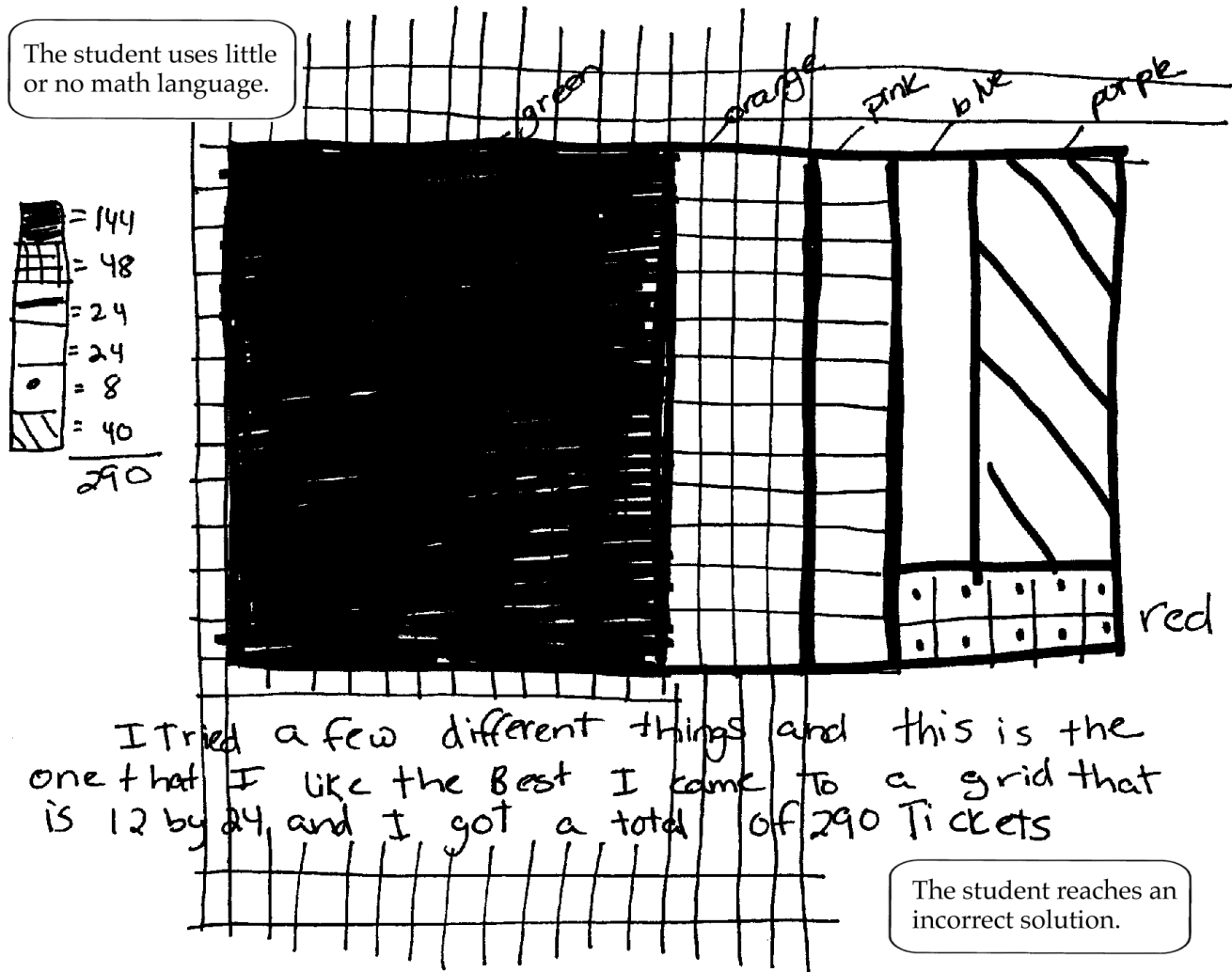
This student worked backwards to find the solution. S/he worked both computationally and with a visual model to verify the solution. Confident use of mathematical language appears throughout the solution. Mathematical representations are clearly labeled (and color coded) and

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work to verify the solution. This student goes on to an extension, which adds depth to the solution. This clever extension gives the opportunity to demonstrate further mathematical understanding.

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Novice



The student fails to recognize the final 8 units represent 60 tickets rather than 8.

This approach may have worked.

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Apprentice

The process the student uses is clear.
The reasoning behind this is unclear.

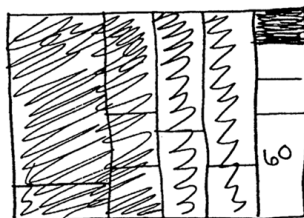
Distribution Dilemma

what I was asked to do
was to find out how many
tickets did Steve have when he
began to give the movie tickets away
what I already know is
on Friday he gave out $\frac{1}{2}$ of his
supply, on Saturday $\frac{1}{3}$ of what was
left, Sunday $\frac{1}{4}$ of the remaining amount.
Monday $\frac{1}{5}$ of what was left today
he gave away $\frac{1}{6}$ of what remained,
also Steve had 60 tickets
left.

what I did to get my answer
is I made a 12×12 Boy on
gide paper. then I cut the
boy in $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ then $\frac{1}{6}$.

then he had 60 tickets left.
then every 20 squares I marked.
then there were 6 marks
So I did $60 \times 6 = 360$
Then I added 11 and I got 372.
So Steve started with
372 tickets.

the Reason for my Decision was
because this was the only
way I could make it work.



$$\begin{array}{r} 60 \\ \times 6 \\ \hline 360 \end{array}$$

Steve started
372 with 372 tickets

The student makes a
computation error.

The student reaches an
incorrect solution.


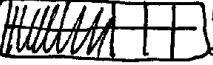
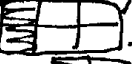



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Practitioner

Distribution Dilemma

The student explains his/her approach and reasoning behind decisions.

We were asked to find out how many tickets did Steve have to begin with. We knew how many tickets he gave out on each day, and how many tickets he was left with. The one piece of information that was really important was that there was 60 tickets left.

We figured out the answer by drawing a diagram/fraction model. We used graph paper, and drew a rectangle with 12 boxes in it, because 12 is a common denominator for 2, 3, 4, and 6, which are the denominators of the other fractions.  Then I colored in the fraction of the rectangle it says for each day. Friday Steve gave out $\frac{1}{2}$ of his supply  Saturday he gave out $\frac{1}{3}$ of what was left  Sunday he gave out $\frac{1}{4}$ of what was left  Monday he gave out $\frac{1}{3}$ of what was left . Sometimes we had to divide the last 2 boxes into 6 so we could color in $\frac{1}{6}$ of what was left . That made me realize that

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Practitioner

The student uses accurate and appropriate math language to communicate.

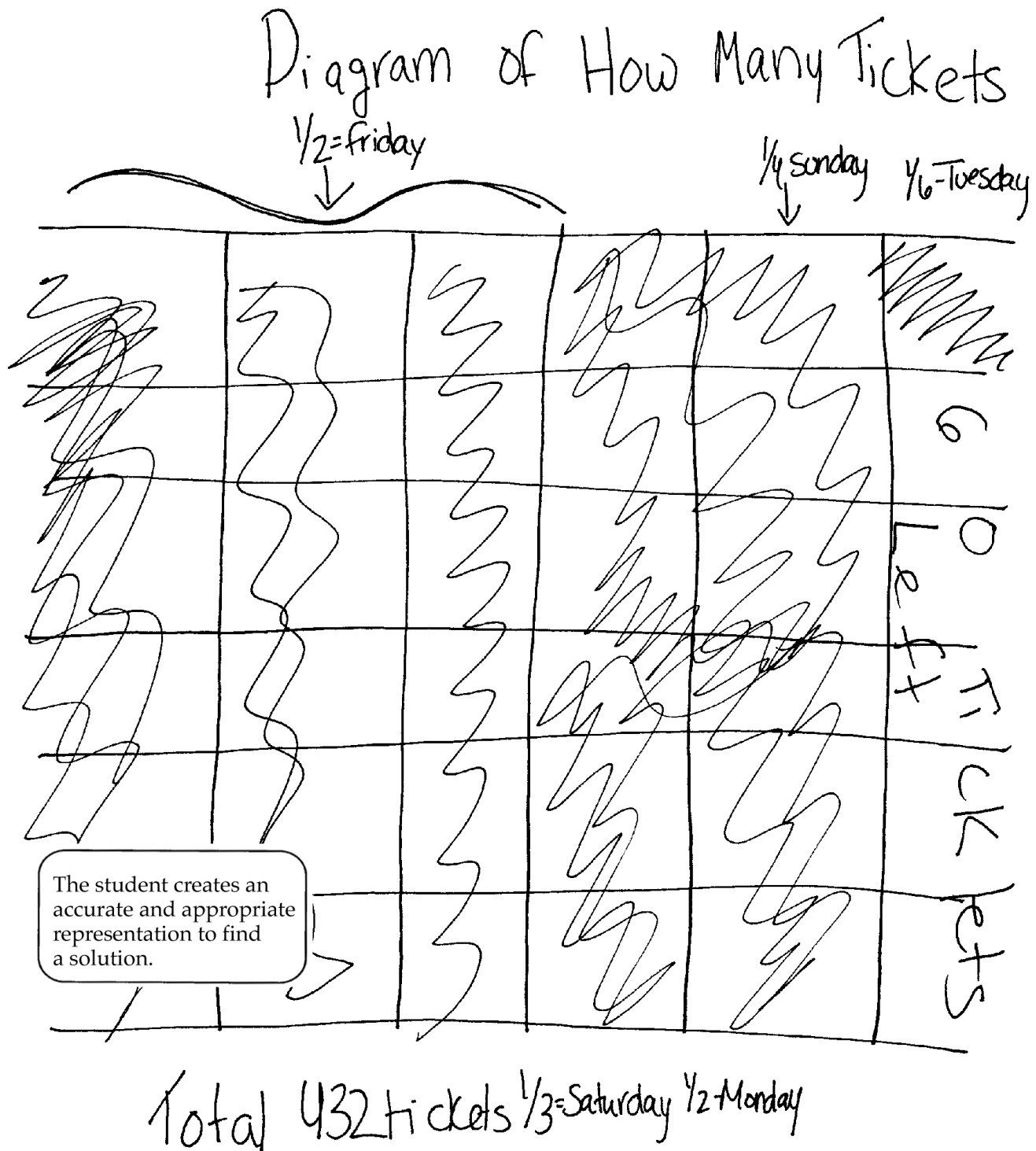
5 boxes equal 60. After thinking about that, we decided to divide each of the twelve squares into 3 like we had to do for the other 2 squares. Then we multiplied 12 by 3 because each of the smaller rectangles in the 12 whole boxes equal 12, and 3 because we divide each box into 3. That answer is 36 which tells me how much each whole box is worth, and then we multiply 36 by 12 because each whole box equals 36, and there are 12 whole boxes, which gives me the answer which is 432 tickets.

This applies to other math I have done, because to figure out the answer I have to use a fraction model to figure out the answer, which I had to do for worksheet 68B.1.

The student obtains a correct solution.

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Practitioner



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Expert

Distribution dilemma

The student explains approach and reasoning.

In this task, a boy, Steve, had given tickets away for the pizza parlor opening. He sold all but 60 tickets in 5 days. We were asked to figure out how many tickets he began with.

1. On Friday he gave half of the supply.
2. On Saturday, he gave $\frac{1}{3}$ of remaining tickets
3. On Sunday, he gave $\frac{1}{4}$ of the rest
4. On Monday, he gave $\frac{1}{5}$ of what was left
5. On Tuesday he gave $\frac{1}{6}$ of what remained
6. He had 60 tickets left

What I did was to work backwards. I started with 60 and divided it into 5 pieces because before he gave $\frac{1}{6}$ away. He had $\frac{6}{6}$ of what remained. how there is $\frac{5}{6}$, $5 \times \frac{12}{60}$

Then I added 12 to 60 because that added piece would make $\frac{6}{6}$ of the remaining = 72 tickets

I used this method to find out the rest of the tickets he sold. On Mon he sold $36(2, \frac{36}{2} = 18)$
on Sun - he sold $36(\frac{36}{3} = 12)$
on Sat. he sold $24(\frac{24}{4} = 6)$
on Fri he sold $216 \rightarrow$
 $(216 + 216 = 432 = \text{total tickets!})$

Accurate and appropriate math language is used.

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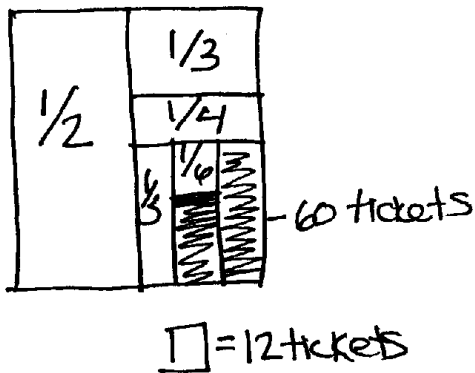
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so what If, tickets would have cost \$5, then the theater lost a lot of money right? Exactly $5 \times 432 = 2160$ dollars. But, depending on how long a movie that these people will watch with their tickets they're going to buy some refreshments. Let's say the average movie is 1 hr 30 min. People would probably buy something like a large soda, (\$2.00) a box small candy, (\$1.25) and a small popcorn (\$1.25) that equals $2.00 + 1.25 + 1.25 = 4.50$. That means for each ticket they would lose 50¢. $(5.00 - 4.50 = 50¢)$ $432 \times .5 = \$216$. That means that they really only gave out $43\frac{1}{5}$ tickets. ($216 \div 5 = 43\frac{1}{5}$).

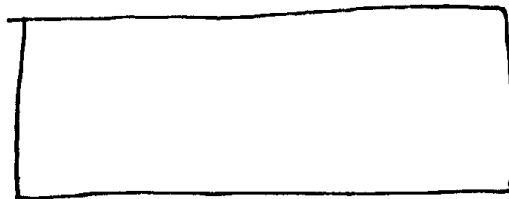
The student explains his/her reasoning.

Exemplars

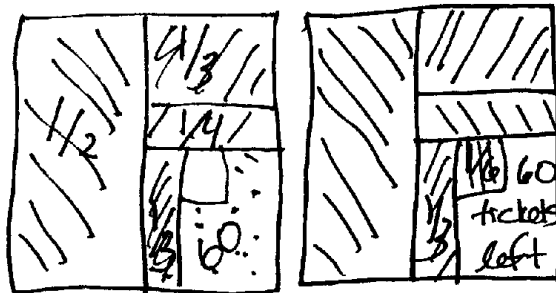
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Accurate and appropriate diagrams are used to communicate the approach and solution.



The student uses diagrams to verify his/her solution.



$$\begin{array}{r} 12 \\ 5 \overline{) 60} \end{array}$$

$\square = 10$
tickets

to do this I divided $(60 \div 5 = 12)$
then to find the area of the square
I multiplied $(12 \times 3 = 36)$. $(36 \times 2 = 72)$.

$\square = 12 \text{ tickets}$

$$\begin{array}{r} 12 \\ \times 36 \\ \hline 432 \end{array}$$

Exemplars

Expert

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