

Candy Dilemma

I bought a box of candy for myself last week. However, by the time I got home I had eaten $\frac{1}{4}$ of the candies. As I was putting the groceries away, I ate $\frac{1}{2}$ of what was left. There are now 6 chocolates left in the box. How many chocolates were in the box to begin with?

Be sure to show and explain all of your reasoning.

Exemplars

Grade Levels 6 - 8

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Context

My sixth grade class was beginning to work with fractions. I knew they had some experience with fractions in fifth grade, but I wanted to assess their flexibility in working with fractions in a problem-solving situation. Sometimes in my class I give an assessment piece and I do not encourage or discourage kids from discussing the problem. Usually if it is a problem that is very challenging, most kids will need to discuss the problem with another peer to clarify their own thinking or strategy. I know that most of the time these problems would not have been solved as nicely - if at all - if they had to solve them on their own. However, I try to make my classroom mirror real situations where problem solving is not usually done in a vacuum. Other times, I want to find a problem that I feel is challenging and yet at a level that can be solved by most of the students without peer collaboration. "The Candy Dilemma" is a problem I gave to my class to solve on their own without discussion with peers.

What This Task Accomplishes

This task allows me to assess my students' ability to problem solve on their own. It is a problem that most kids can at least begin to approach individually.

What the Student Will Do

Some students began by drawing pictures. Other students used a guess-and-check strategy. Some tried to work backwards and were successful.

Time Required for Task

45 minutes

Interdisciplinary Links

None

Teaching Tips

Candy Dilemma

Exemplars

Some students came up with the incorrect solution of 24 candies fairly quick. They thought $1/4 + 1/2 = 3/4$ so the six candies left must be $1/4$ of the box. They had not drawn a diagram or really thought about their thinking. I encouraged them to "check" their solution by drawing a diagram - some realized their error when they did check. This is a good problem to ask students to show more than one strategy.

Suggested Materials

- Graph paper (Some students may want to use.)
- Circle graph (I had students who wanted to use a compass and protractor.)

Possible Solutions

The six candies represent $1/2$ of $3/4$ of a box = $3/8$. Therefore, each $1/8$ of the box has two candies. This makes 16 candies in a full box.

Benchmark Descriptors

Novice

This student is using inappropriate concepts to solve the problem. S/he is trying to add $1/2$ and $1/4$ by finding a common denominator. The explanation is incomplete as to how s/he found the answer to be 24 candies, except to assume they thought $3/4$ of the box of candies were eaten and $1/4$ or six candies were left. The strategy of adding $1/4$ and $1/2$, and thinking that is the amount of candy eaten, is an error in mathematical reasoning and a strategy that is not helpful in solving the problem. There is no use of mathematical representation.

Apprentice

This student's solution is not complete (s/he gives the fraction of the box that is left uneaten) indicating part of the problem is not understood. The student uses a strategy that is partially useful leading some way toward a solution, but not to the full solution of how many candies are in the box. There is evidence of mathematical reasoning, but the student did not take the solution to the end. There is some use of mathematical representation (the circle graph and the rectangular representation of the box of candies) and mathematical terminology (degrees, fractions). (There were students who were able to follow through with the idea of a circle graph to successfully solve the problem. These students I rated as Expert because of their ability to connect central angles and fractions.)

Practitioner

This student uses guess and check as a solution. Through guess and check, it is clear the student understands the major concepts necessary for the solution. There is a clear explanation and the student uses mathematical representation as well as mathematical terminology.

Expert

This student has a deep understanding of the problem including the ability to use beginning

Exemplars

algebra concepts (without having taken any formal algebra course) as an efficient and sophisticated strategy leading directly to a solution. There is a clear and effective explanation detailing how the problem was solved. The student includes a diagram that shows a possible second strategy to solve the problem. There is precise use of mathematical terminology and notation ($\frac{1}{2}$ of $\frac{3}{4}C$ is $\frac{3}{8}C$; reciprocal; $\frac{48}{3} = 16$).

Exemplars

Novice

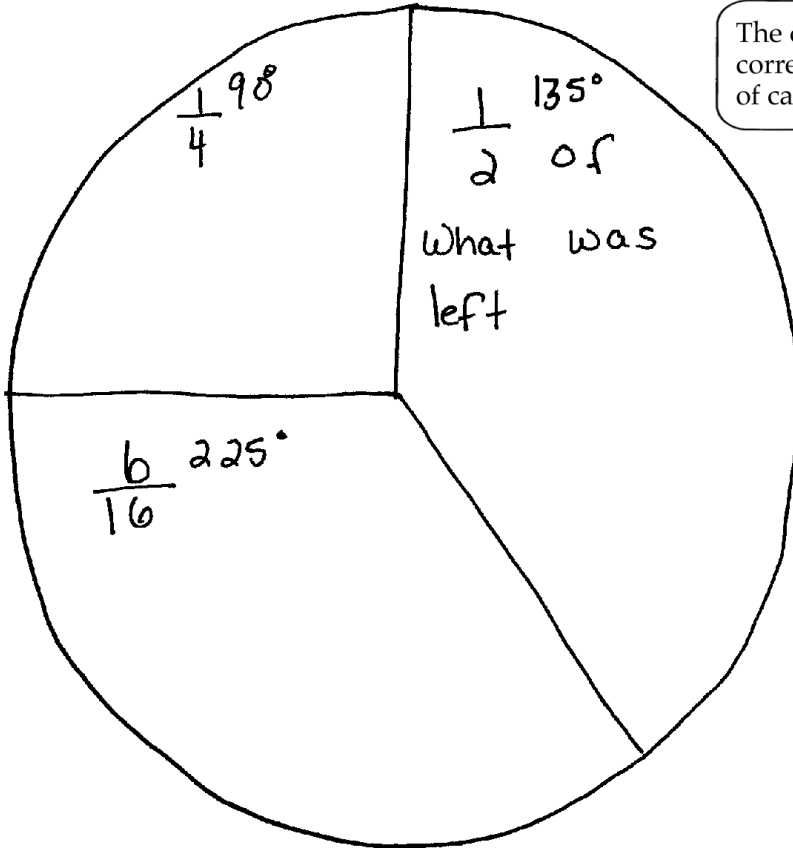
No mathematical reason for the answer of 24. No reason for finding the LCM of 4 and 2.

The first thing I did was figure out the LCM of 4 and 2. I found that the answer is 24. So the answer is 24 candies.

Exemplars

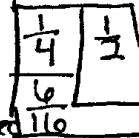
Apprentice

I divided 360° by 4 and got 90° so my $\frac{1}{4}$ is by 90° degrees then I divided the rest of the degrees in half then the rest was the answer and it is 225° or



The circle graph is correct, but no number of candies is given.

I did it this way because ↓
 I made a square or 4×4
 and then that equals 16 so
 I divided 16 by 4 to
 get fourths then there
 were 12 left so I divided 16
 in half because you were supposed to
 find one half of the rest then there's
 $\frac{6}{16}$ left in the box.



The student is finding the fraction of what is left in the box, but misunderstood the problem, which was to find the number of candies in the box.

$$\frac{6}{16}$$

Exemplars

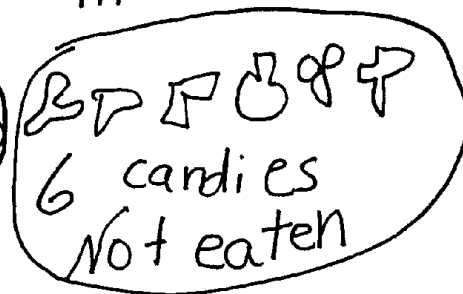
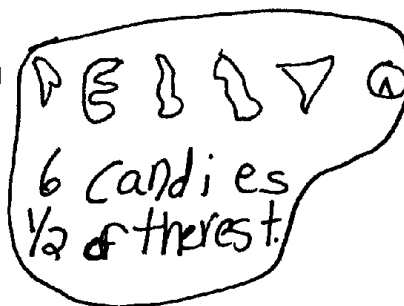
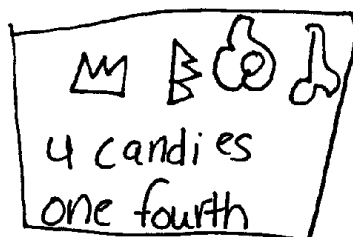
Practitioner

As the student explains his/her strategy – it shows understanding of the problem.

Assuming you bought a full box of candy there were 16 candies to begin with.

This is because $\frac{1}{4}$ of 16 is 4, so you subtract 4 from 16 and get 12. $\frac{1}{2}$ of 12 is 6 and $12 - 6$ is 6 and you had 6 candies left after putting the groceries away.

I found this out just by looking at the fraction and then guessing. So by trial and error I found out that there were 16 candies



Not a strong diagram, but it is appropriate to the problem.

Exemplars

Expert

First I knew I had to figure out how many candies were in the box to begin with. The number of candies in the box would equal the letter C . I ate $\frac{1}{4}$ of C . This left me with $\frac{3}{4}C$. Then I ate $\frac{1}{2}$ of $\frac{3}{4}C$. This left me with $\frac{3}{8}C$. I knew that $\frac{3}{8}C$ equaled 6. Some how I had to figure out what C alone would equal. I know that if I multiplied $\frac{3}{8}$ times its reciprocal of $\frac{8}{3}$ it would equal 1 or just C alone. Then I would have to multiply $6 \times \frac{8}{3}$ to keep every thing equal. So $6 \times \frac{8}{3} = \frac{48}{3}$ which simplifies to equal 16.

So

16

Sophisticated mathematical notation.

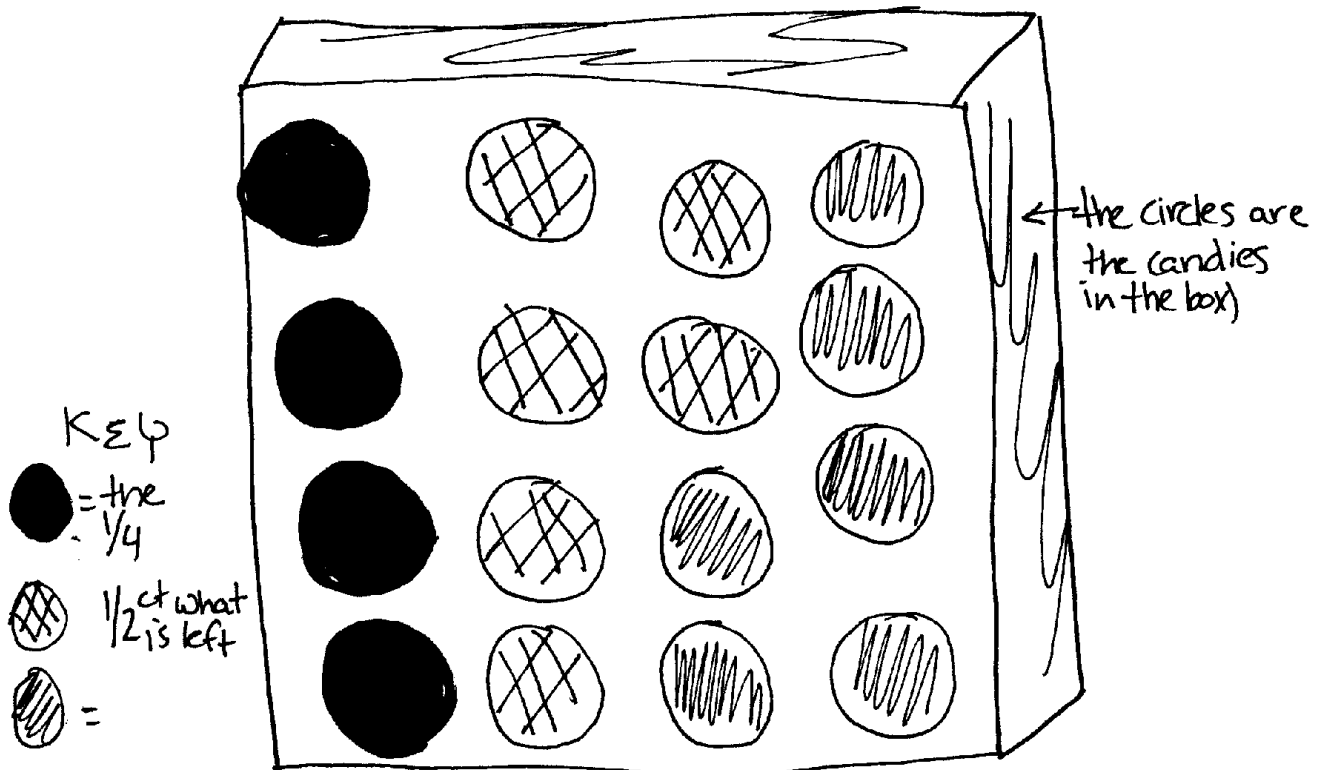
Efficient solution and use of algebra.

is the answer

Exemplars

Expert

CANDY COUNT



Not a very sophisticated diagram, but it does verify the solution.

this way of explaining dose not match my write up.)