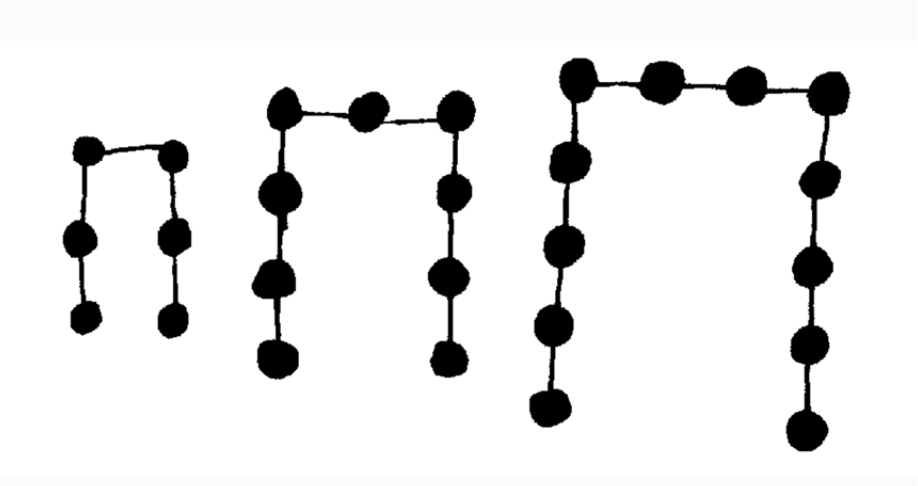
Bridges

You are given the first 3 bridge designs.

Look at the number of pegs needed to build each bridge. Using this pattern, decide how many pegs are needed for the 10th bridge design. Be sure you show all your work in an organized way so anyone looking at your solution will be confident you correctly solved the problem. Talk about at least one pattern that you notice. If you can, look for a generalized equation that would help to find the number of pegs needed in any sequenced bridge design.

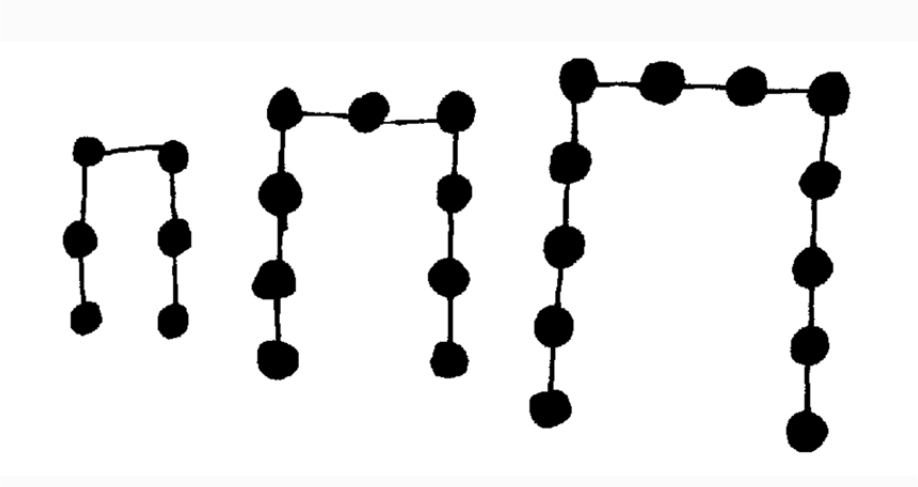


Grade Levels 6 - 8

Bridges

You are given the first 3 bridge designs.

Look at the number of pegs needed to build each bridge. Using this pattern, decide how many pegs are needed for the 10th bridge design. Be sure you show all your work in an organized way so anyone looking at your solution will be confident you correctly solved the problem. Talk about at least one pattern that you notice. If you can, look for a generalized equation that would help to find the number of pegs needed in any sequenced bridge design.



Context

The sixth grade text we use introduces students to algebra by giving them an equation and having them fill in a chart for X and Y. They also give a chart of values and ask the students to come up with the equation. I wanted to build a context that would assess students' depth of knowledge of the use of charts to record data and look for patterns and come to generalizations. Notice the problem prompts the students to try for generalizations.

What This Task Accomplishes

The task allows students to see mathematical patterns in context. Just about all students will be able to get involved with the task and many will be able to see a simple pattern. A few will be able to see the broader pattern by manipulating the numbers and come up with a generalization. Still fewer will be able to actually see the pattern in the actual design of the

bridge. The problem is also a good example of the use of charts to organize information.

What the Student Will Do

All students started by looking at the bridge designs and counting the number of pegs for each of the three given bridges. Some recorded it right below the designs, while others realized that a chart would help to record these and the next seven bridges. Some students started making the next seven bridges, and as they did, began to see the pattern of adding one more peg to each leg and one peg across the top. Some students who went too quickly to the chart missed this connection. Most students saw and commented on the fact that each successive bridge has three more pegs than the previous bridge. Others were able to link the bridge sequence number to the number of pegs using an equation.

Time Required for Task

45 minutes

Interdisciplinary Links

This problem could be given with a unit on bridges from different cultures or different eras. It also could go along with a science unit on bridge constructions.

Teaching Tips

Students should be aware of some beginning algebra ideas. They should have been exposed to some "what's my rule" type of problems where you give them a sequence of numbers and they try to come up with the algebraic rule that will find each number.

Suggested Materials

- Rulers
- Graph paper
- My students also know that there are cubes available to use at any time.

Possible Solutions

The 10th bridge will have 33 pegs. One pattern is that each successive bridge increases by three pegs. The generalization is that the number of the bridge in sequence multiplied by the three plus three is the number of pegs in that bridge.

$3B + 3 = P$ or $3(B + 1) = P$

Benchmark Descriptors

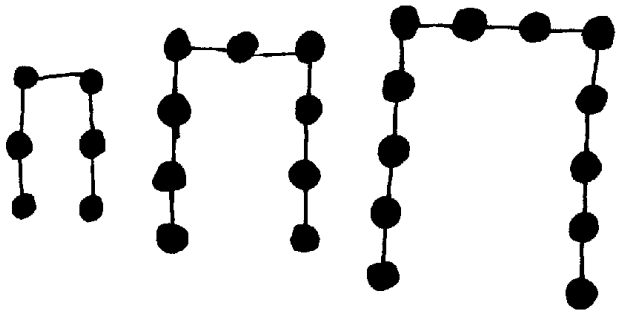
Novice
There is no solution. This student could count the number of pegs in the first three bridges and put that information in a chart, but could not see how the other bridges could be recorded. There is also no evidence that the student could construct the other bridges.

Apprentice
This student's solution is not quite complete. The problem asked the students to talk about one pattern that they observed. The student used a strategy that was useful to get the number of pegs in the 10th bridge, but it did not help the student identify a pattern. The use of a chart shows some evidence that the student was reasoning mathematically, however, there is an incomplete explanation.

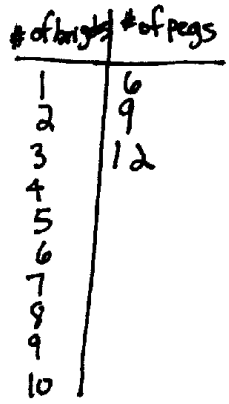
Practitioner
This student has a broad understanding of the problem and the major concepts necessary for the solution. S/he has correctly identified the number of pegs in the 10th bridge and has noticed that the number of pegs goes up three each time. S/he also connected the design to the number pattern and noticed where those three pegs were being added. S/he noticed groups of six in the 10th bridge. A Practitioner is also a person who attempts to find a generalization and writes an equation that says: B = the number of pegs in previous bridge and writes this equation, B + 3 = pegs in bridge.

Expert
This student has a deep understanding of the problem including the ability to identify the appropriate mathematical concepts. By reaching a generalization, this student employed refined and complex reasoning. There is a clear and accurate explanation and the mathematical language and representation clearly communicates the student's reasoning. This particular student also connected the equation to the actual design. Some Experts looked at their chart and manipulated the numbers until they came up with an equation.

Novice



The student is unable to find a solution for 10 bridges.

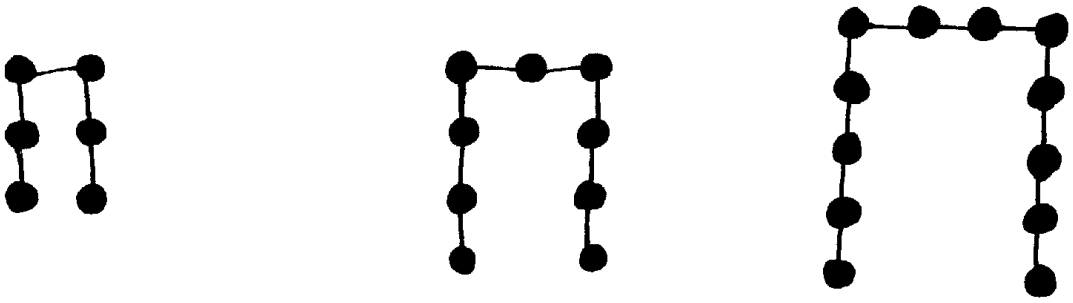


The representation is labeled, but incomplete.

Apprentice

The math representations are labeled and appropriate.

The student is able to determine a solution for 10 bridges, but stops.

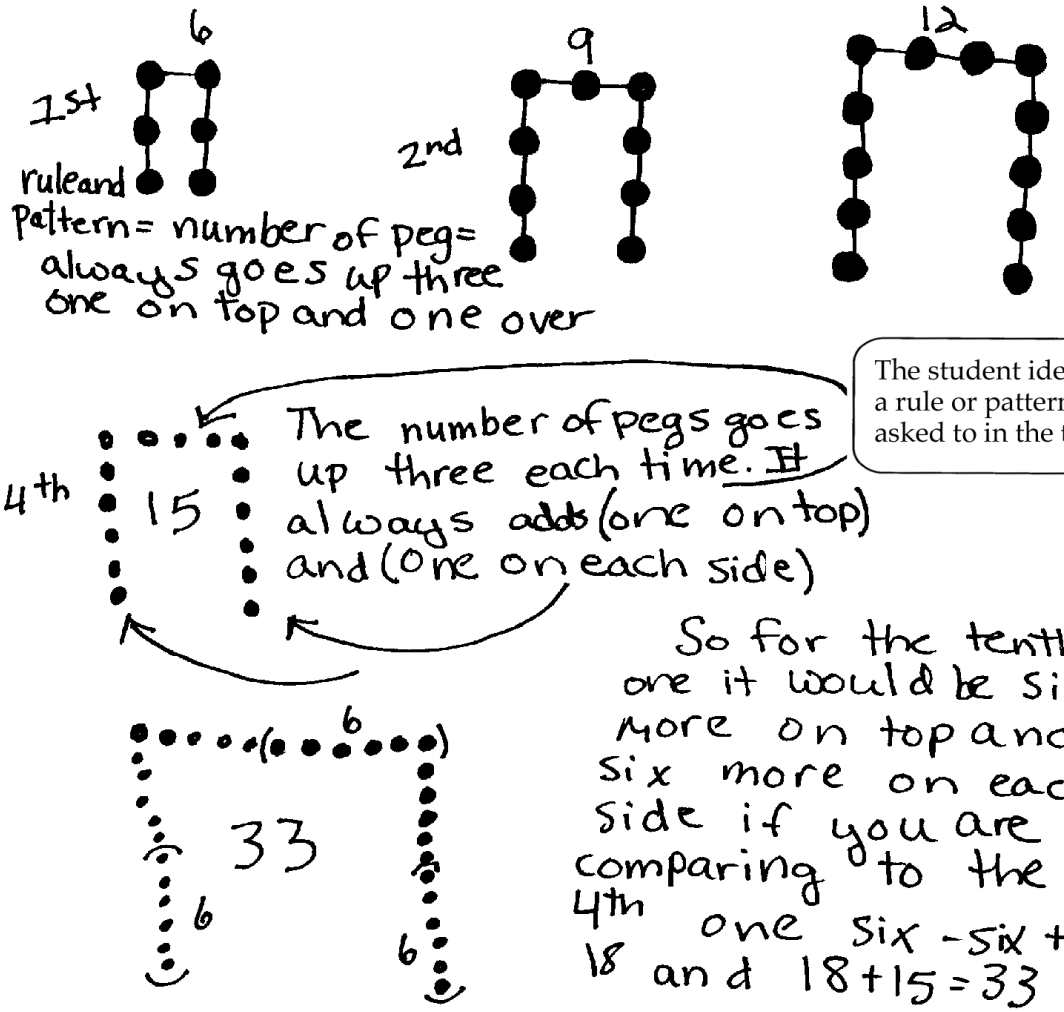


bridges	# of pegs
1	6
2	9
3	12
4	15
5	18
6	21
4	24
8	27
9	30

Little or no math language is used.

# bridges	# pegs
10	33

Practitioner



The student identifies a rule or pattern as asked to in the task.

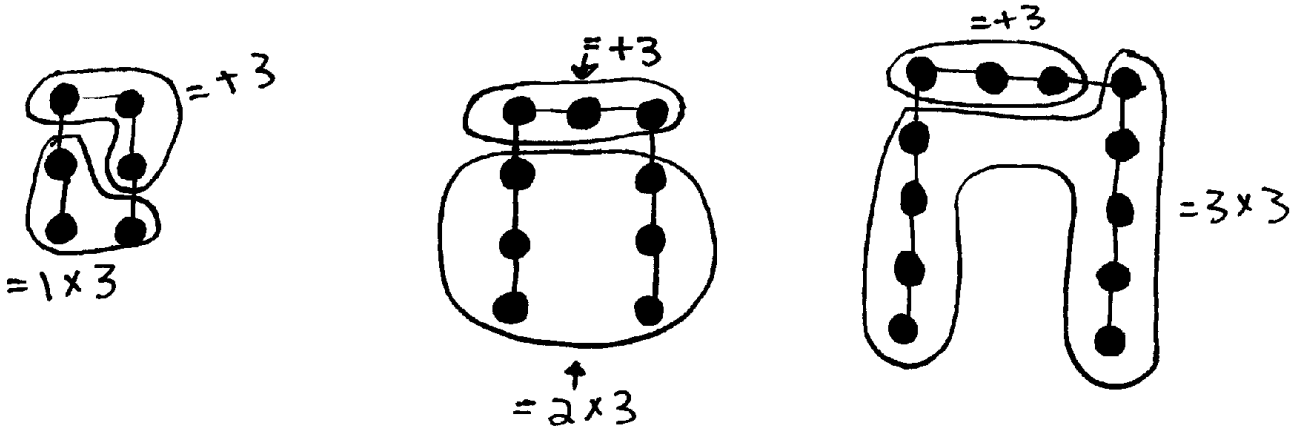
Representations are accurate and appropriate.

# Bridge	# pegs
1	6
2	9
3	12
4	15
5	18
6	21
7	24
8	27
9	30
10	35

The student is able to determine a solution for 10 bridges.

Expert

Representations are labeled, accurate and appropriate.



x # of bridge	y # of pegs
1	6
2	9
3	12
4	15
5	18
6	21
7	24
8	27
9	30
10	33

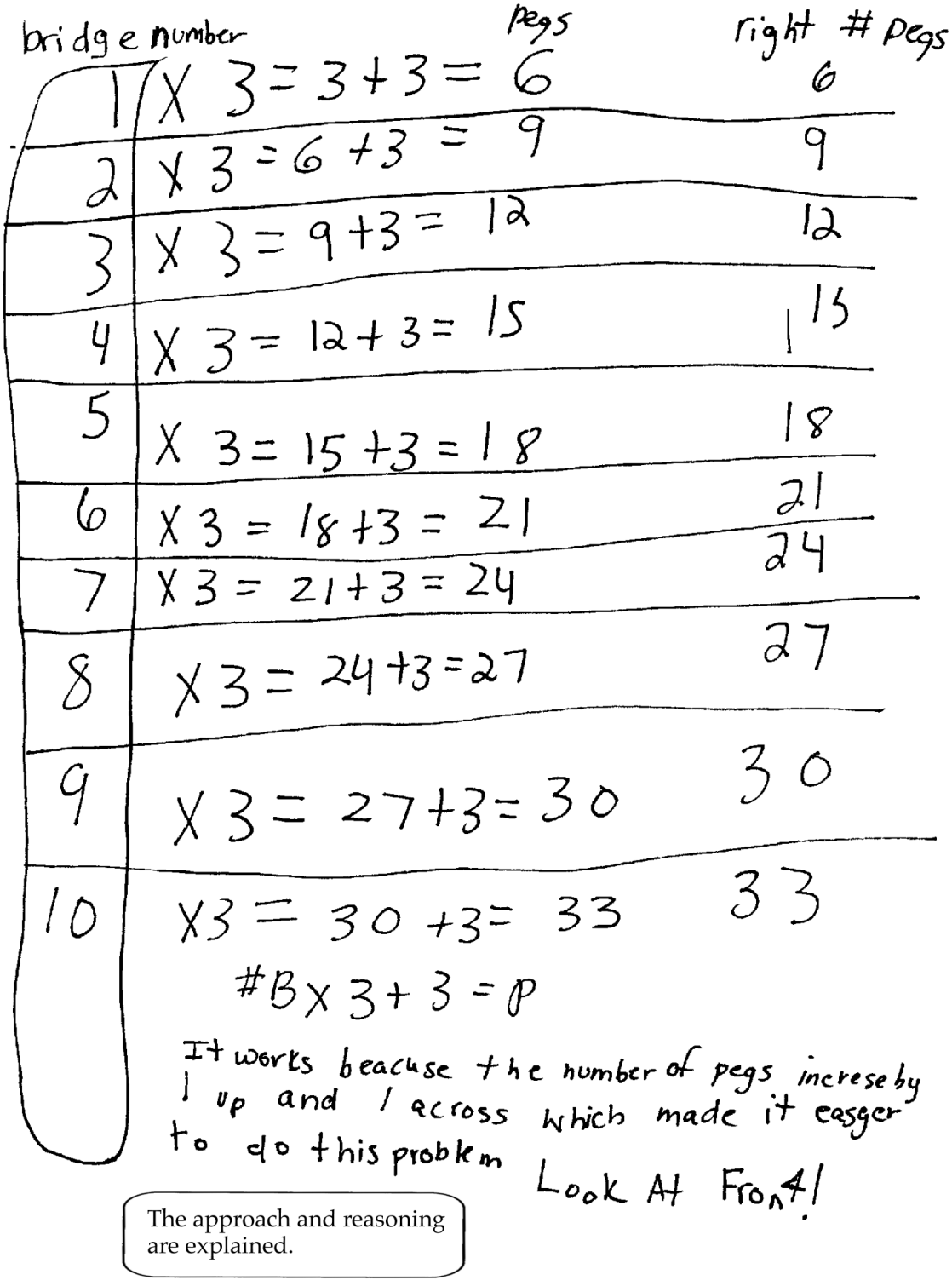
The number of pegs in the 10th bridge = 33. The equation is $(x-3)+3=y$. I know that because each bridge is bigger then the other by 3 pegs.*

* see bridges for more information

Sophisticated math language is used to communicate.

The student generalizes the solution for any number of bridges.

Expert



The approach and reasoning are explained.