

## Dog House

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## Grade Levels 6 - 8

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#### Context

This task is taken from the *NCTM Curriculum and Evaluation Standards*. It can be found under *Standard 13: Measurement* on page 118.

The class is a multi-age sixth-seventh group of 56 heterogeneous students, working in cooperative groups.

We had been doing a lot of measurement activities and I was looking for a task that would extend their thinking to a new application where they would have to resort to models since the actual dimensions were so large. Asking for the largest house possible, necessitates trying a variety of models and calculating the volume of each. This encourages the use of a table or chart for organization. It also calls on students to consider the practicality of some of their designs.

#### What This Task Accomplishes

This task allowed much mathematical communication as I had the students working in groups of three or four. They used grid paper to represent the sheet of plywood and many converted from centimeters to decimeters in order to have a one-to-one correspondence. Many of the models were odd shaped and required some research into calculating volume especially by the sixth graders.

#### What the Student Will Do

The groups brainstormed ways to cut the wood and piece it together to make the largest looking structures. Many used grid paper and cut out rectangles 15 x 30 squares and experimented with various possibilities. Many then taped the pieces together to make models. By comparing the volume of each model, they determined the largest house. Some groups then made some judgments that the largest possible shelter would not be appropriate for protecting a dog for one reason or another. As most students own dogs, there was considerable expertise! It became apparent to most groups that they needed a chart of some sort to organize their data once they were done.

#### Time Required for Task

#### Dog House

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# Exemplars

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This task took the better part of a week of 40-minute classes to complete. Some groups worked outside of class time to complete the task on time.

## Interdisciplinary Links

Students had done mechanical drawings and blueprint designs in Technology Education a few weeks beforehand and recalled that training in their work on this task. It could be adapted to create something other than a doghouse based on an interdisciplinary unit of study you might be doing at the time.

## Teaching Tips

Have a lot of grid paper, scrap oak tag (old manila folders) and tape on hand. Having the students work in groups enables them to come up with a wider selection of designs and provides many hands for making the models in less time.

Students would have been content to work on this task for weeks if time allowed. They kept coming up with new ideas for further designs to push for the largest possible design. It became quite a contest as groups posted their current record volume numbers on the flip chart.

There was much discussion about what constitutes a humane dog shelter. Many individuals thought that digging an enormous hole and covering it with the entire piece of wood was the best idea. Their peers disallowed this option.

## Suggested Materials

- Calculators
- Rulers
- Grid paper
- Oak tag (old manila folders)
- Scotch tape

## Possible Solutions

Using a 10 cm strip to cut four 5 cm x 75 cm legs to support the rest of the sheet (like a table) will give a volume of 3,262,500 cm<sup>3</sup>. This is about the largest reasonable design found by this group.

Cutting the sheet in half to form two 150 cm x 150 cm halves and leaning them together (tent style) will give a volume of 1,687,555 cm<sup>3</sup>.

## Benchmark Descriptors

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# Exemplars

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## **Novice**

This group tried only one design. They drew a diagram of the completed house, but did not label the pieces on the sketch of the full sheet of plywood. Their diagrams lack any meaningful labels - there are no units and only sketchy numbers. There is no evidence of calculating the volume of this design. There is no discussion as to how they selected this to be the largest possible house. When prompted, their second attempt accomplished little.

## **Apprentice**

This group did a comparison between two designs, but stopped there. The two designs were similar (both were tent-like and had only one cut of the wood). The calculations are accurate. They speak of "area" when they mean volume in paragraph three. They do offer diagrams of their models, but omit any measurements. Their offering of a graph does little to enhance one's understanding of their work. They attempt no observations, connections or generalizations.

## **Practitioner**

This group constructed three models from sketches they had drawn. While they do not document it well, they had done some rough calculations to discover that none of the traditional designs would approach their numbers for volume and they were going for largest possible volume regardless of doggie comfort! Their consciences did catch up with them as time went on, as witnessed by their decision to only build number one. Their calculations are correct and their reasoning was good. They have a good variety of math language used accurately throughout. This is a group that really benefited from working together, as they would challenge each other to try for even greater volume. They were completely involved for days. Unfortunately, their 3-D models did not copy well. They were made from oak tag and colored beautifully.

## **Expert**

This group designed four houses of various shapes and sizes all based on criteria established at the outset regarding dog comfort. Doghouse B and C show their understanding of difference of volume based on subtle measurement changes. More mature students might have made an observation based on these models and their choice of design D. As they approach cubic design, the volume continues to increase. This concept is demonstrated nicely in their organizational chart. They move with ease through the metric conversions to suit their needs. They use a variety of good math language including symbols throughout the solution. They attempt a connection to their "real world", but it falls short of being mathematical. Their strength is in their explanation of how and why they do as they do and the practicality of their decisions throughout the task.

# Exemplars

## Novice

Roof tiles are  $8 \times 7$   
Door tiles  $8 \times 7$   
Sides  $8 \times 7$   
Back  $8 \times 7$

This student shows limited awareness of the problem. Work lacks labels.

The student does not use all of the plywood.

The part that connects the Roof is  $6 \times 6$   
the width will be  $8 \times 7$  length will be  $8 \times 2$

the dog house has a slanted Roof  
the sides are straight it is a Rectangle  
the doors is round with a half hexagon.

Is this the argest house you can get? yes it is

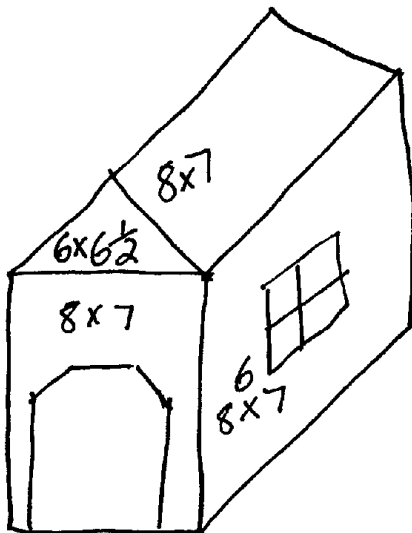
The student neglects to address volume in his/her design.

Maybe a bigger dog house could be built if we built part of it underground. It possibly would be built longer and wider. The sides would be 1 meter long, each side the width would be  $\frac{6}{10}$  of a meter. When you dig the hole it would  $\frac{4}{10}$  deep. The full height would be 1 meter 20c and a half. We would build a ramp 30c long.

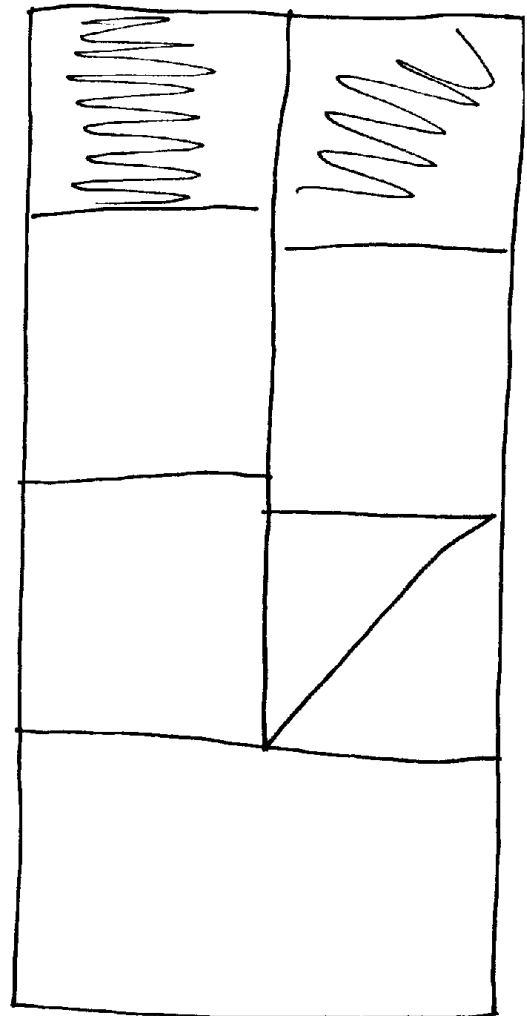
# Exemplars

## Novice

This diagram lacks labels and details.



2  
 $6 \times 6 \frac{1}{2} \frac{1}{2}$   
~~8~~  
 $8 \times 7$



# Exemplars

## Apprentice

The students neglect to explain their reasoning.

For the doghouse task, we had to make the largest doghouse possible out of a piece of plywood,  $300\text{cm} \times 150\text{cm}$ . We tried 2 different models. Our doghouse looked like a tent.

The first model we tried the 2 pieces of wood that we put together to look like a tent was  $150\text{cm} \times 150\text{cm}$ . That was only an area of  $131,250$  cubic cm. The second doghouse we tried, the dimensions of both pieces of wood was  $300\text{cm} \times 75\text{cm}$ . The area of that doghouse was  $1,462,500\text{cc}$ .

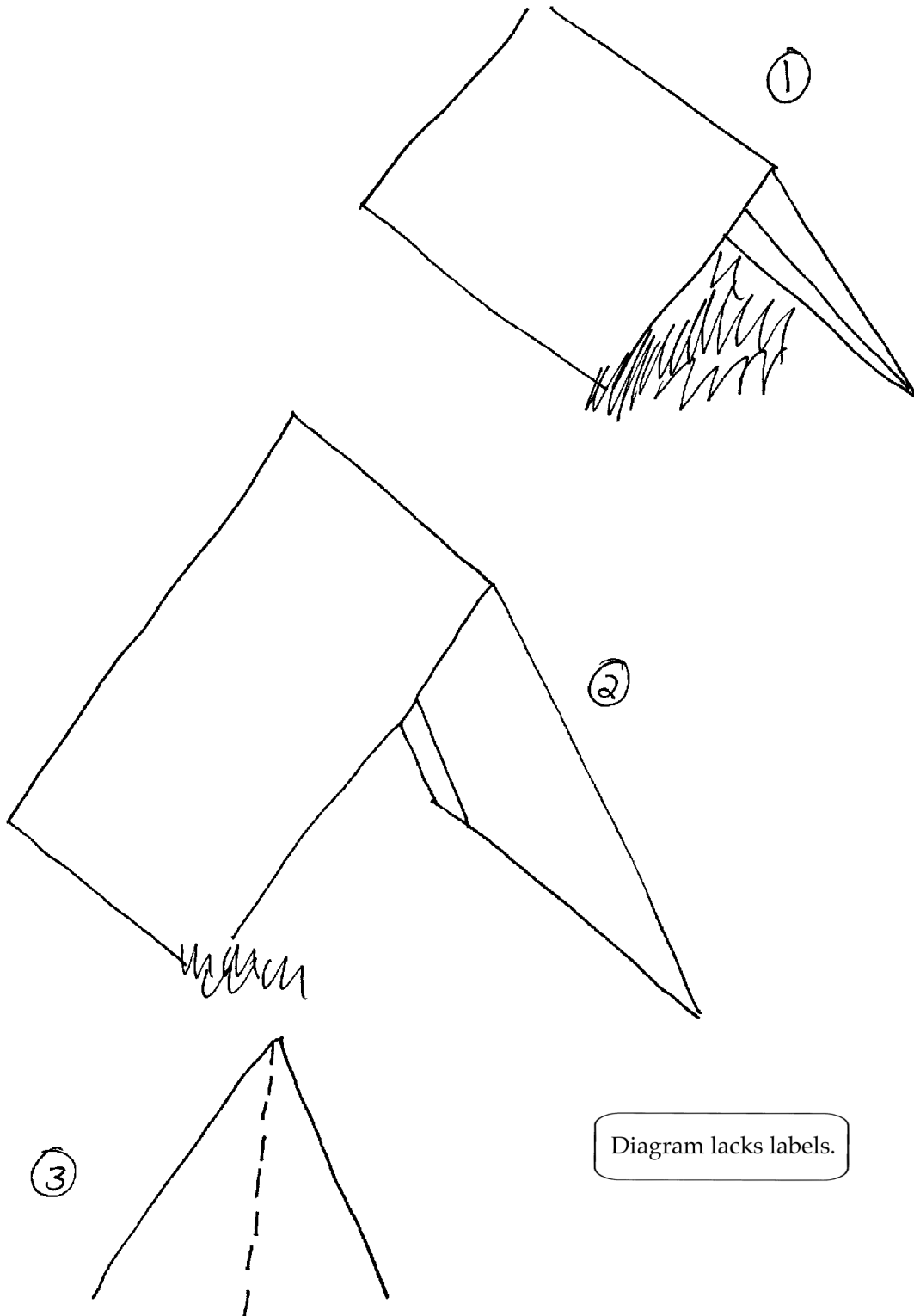
The student explains their approach.

The way we figured out to find the area we made a dotted line down the middle to form 2 right triangles. Then we multiplied height  $\times$  base  $\times$  width. Then we divided it by 2, then we multiplied it by the depth. That gave us an area of  $1,462,500\text{cc}$ .

The students confuse area with volume.

# Exemplars

## Apprentice

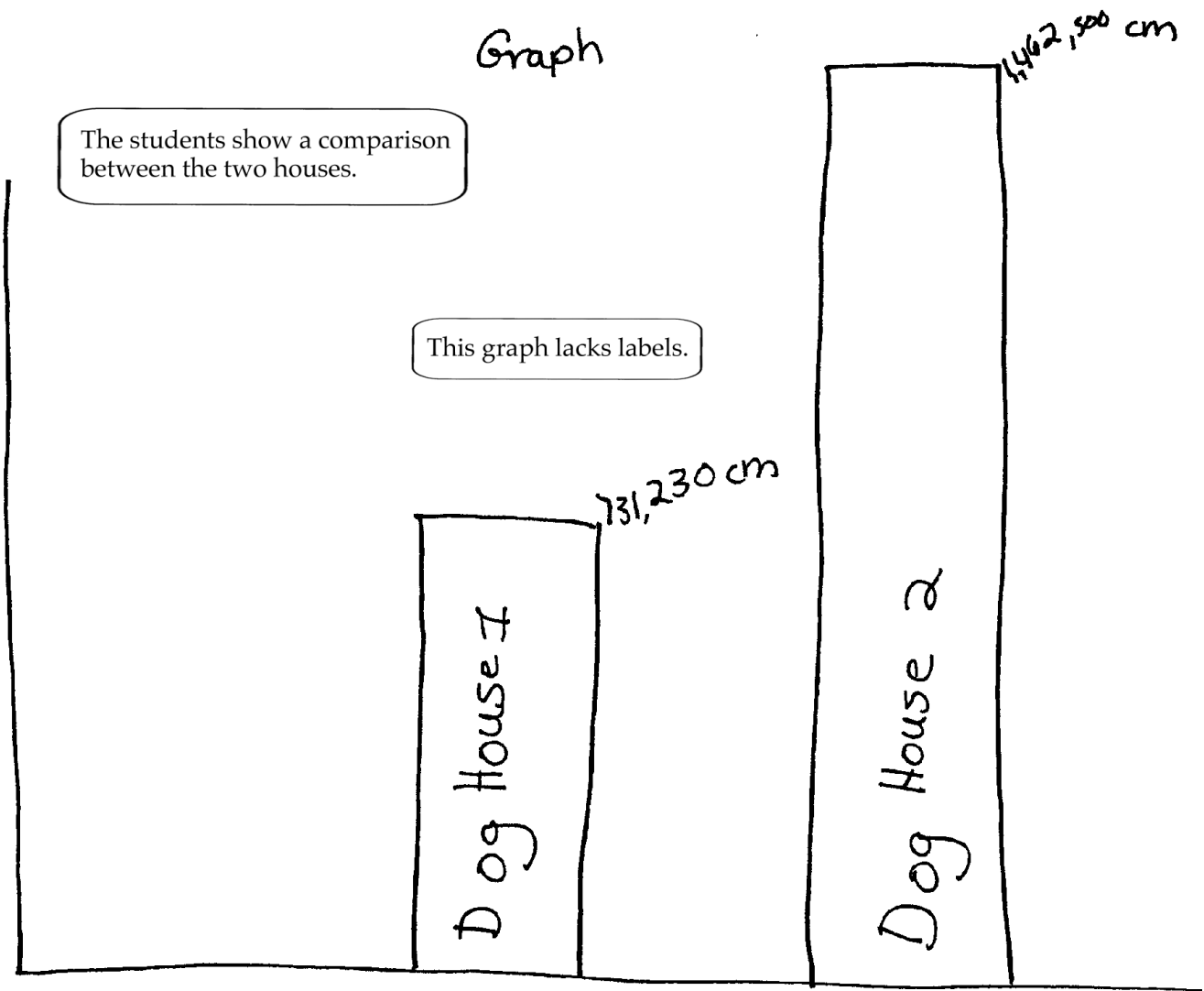


## Dog House



# Exemplars

## Apprentice



# Exemplars

## Practitioner

The student attempts three different designs and evaluates each.

The first dog house is two square pieces of wood leaning up against each other. The house has no ends or bottom.

The second dog house is like a lean two. we cut one strip of wood off the board and used that to support the top.

The third house is like a table with 4 legs. This is our biggest house.

We think number 2 and 3 are mean to your dog because it only keeps out the rain and snow but not the wind. If we were going to build one we would build number 1.

The best way to figure out how to do this would be to draw or build your idea and see which one is the biggest.

The reason we didn't make a regular dog house is because it would be tiny compared to the others as the model shows.

The volume of number one is  $1,687,500 \text{ cm}^3$  we figured that out by multiplying  $150 \text{ cm} \times 150 \text{ cm} \times 150 \text{ cm}$  And we got  $3,375,000 \text{ cm}^3$  And then we divided that by two because it is a equilateral triangle.

The volume of number two is  $3,352,500 \text{ cm}^3$  we got that the same way. we turned the triangle into a square and divided that by two.

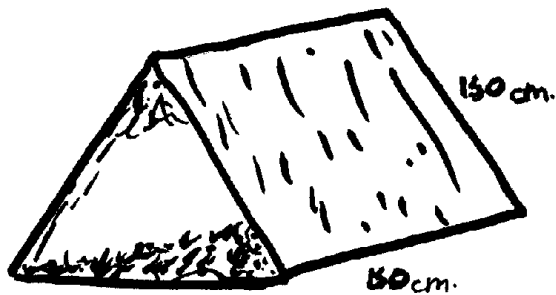
The volume of number 3 is  $3,262,500 \text{ cm}^3$  we got that by multiplying  $75 \text{ cm} \times 150 \text{ cm} \times 290 \text{ cm}$ .

The student explains the approach and reasoning.

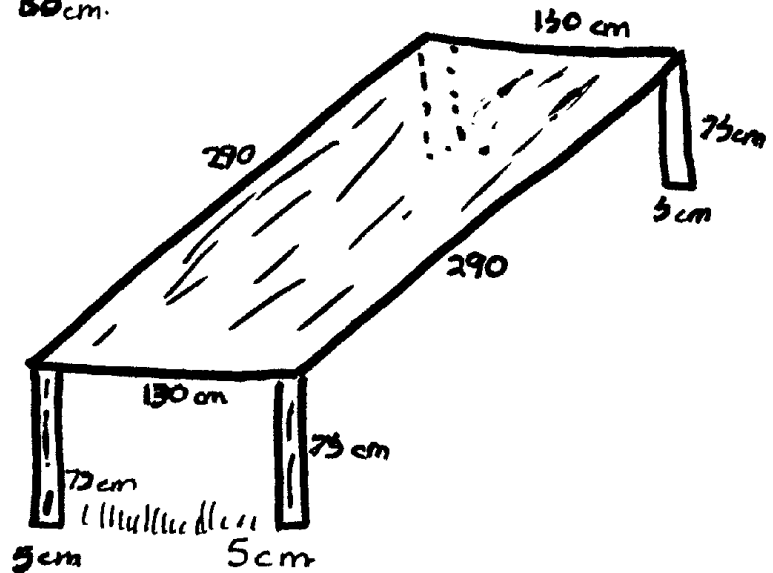
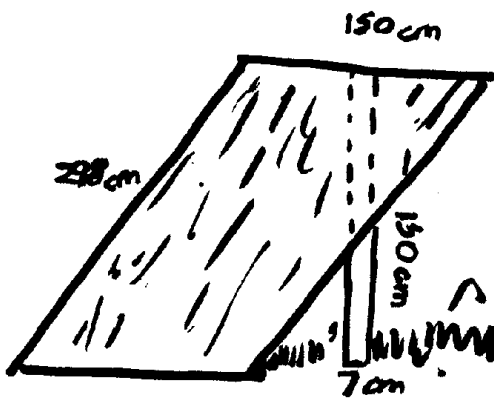
Student uses accurate and appropriate math language.

# Exemplars

## Practitioner



The diagrams are precise and well labeled.



# Exemplars

## Expert

In this problem we were asked to calculate how large the largest dog house we could design with just a 15m x 3m rectangle of plywood. This is how we went about investigating the solution we took a piece of graph paper, and began cutting out the figure of our wood. We decided that one square on the graph paper was equivalent to one square decimeter.

Then we cut the paper into different sizes for the side panels, top panel and back end panel of the structure.

We built many different models of different ways to build the dog house.

We wanted to use all of our wood so that the dog house could be as large as possible.

We decided that the doghouse should have a roof 2 sides and one end we also wanted a small flap so rain and snow wouldn't go into the open front.

The final solution that we came up with was:

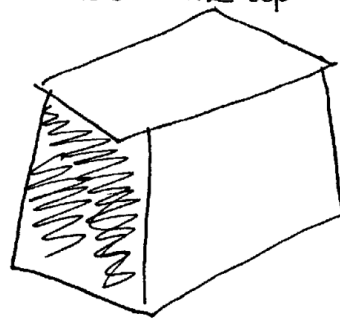
length: 15 dm.

width: 8 dm

height: 8 dm

front flap: 2 dm x 8 dm.

Volume 960,000 cm<sup>3</sup>



Doghouse D

We wanted a wide dog house so we made a flat roof to make it shorter yet wider. This way it would be wide enough for the dog to turn around in.

The student explains the approach and reasoning.

Precise math language is used to communicate.

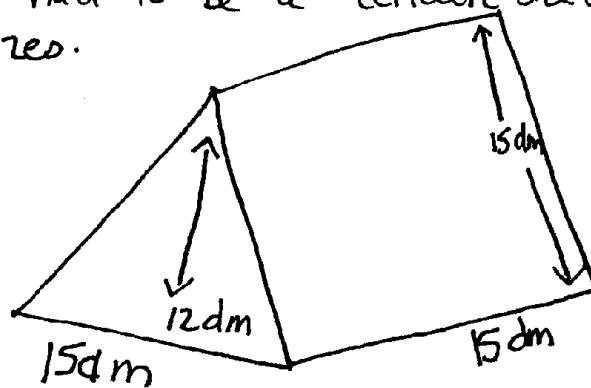
# Exemplars

## Expert

This information would be very useful for all of us if we had to build a real dog house. (3 of the 4 of us have dogs). We would know how much wood we would need and what the dimensions should be.

This pattern would be more difficult if it had to be a certain size or in a range of sizes.

The student evaluates the solution.



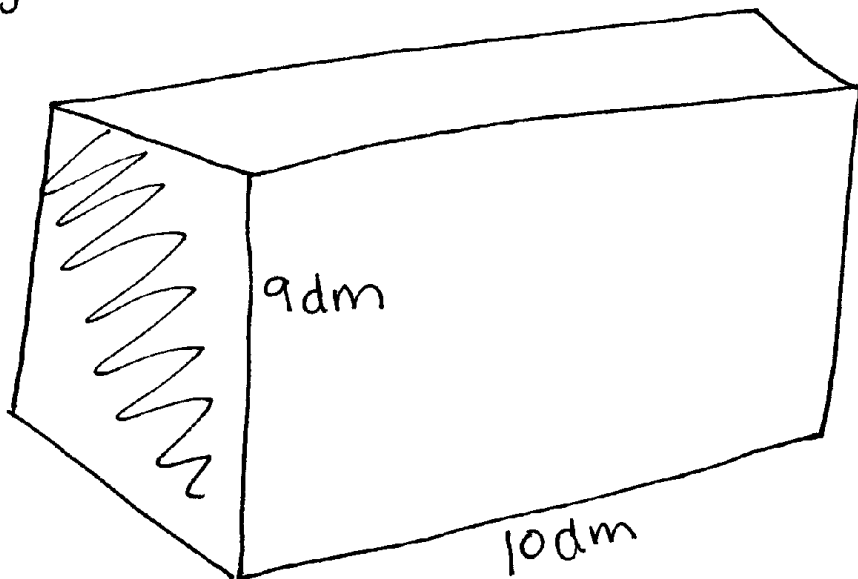
Volume:  
 $818,181 \text{ cm}^3$

None of these structures achieved our standard for building the dog house. They all had less volume than our final solution #1 was good although number 2 had more volume. We wanted to have at least four faces on our dog house. A roof, 2 sides, and an end. That is why we ruled out this option. Also, it had less volume than our solution.

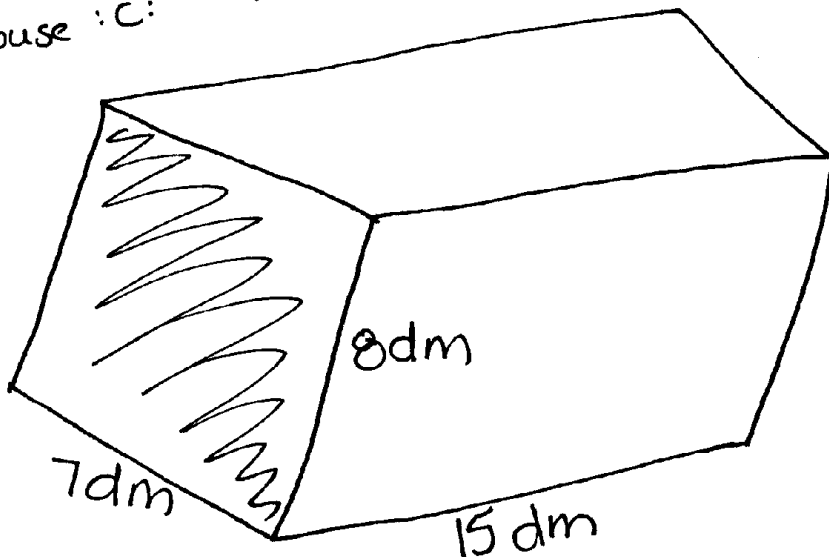
# Exemplars

Expert

these are 3 of the other designs that we tried  
Doghouse 13      Volume: 810,000 cm<sup>3</sup>



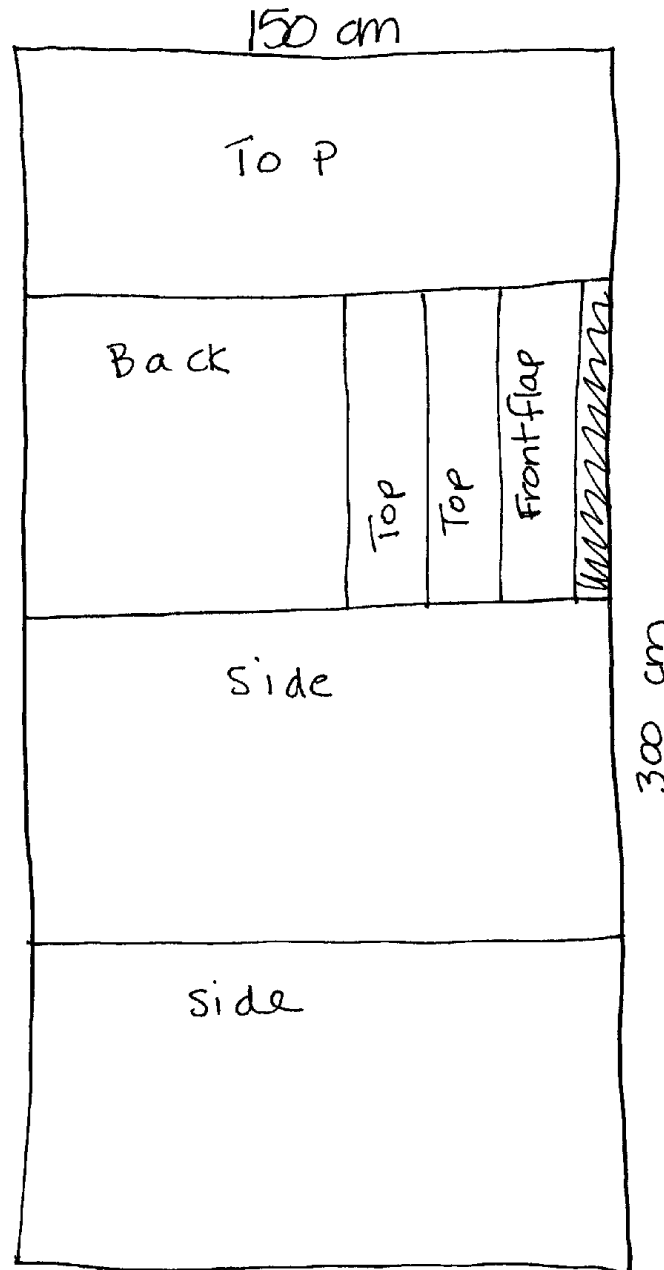
Dog house :c:      Volume: 870,000 cm<sup>3</sup>



# Exemplars

## Expert

these are the  
pieces we used



# Exemplars

## Expert

Doghouse Chart

Doghouse	Length	width	height	volume
A	150 cm	150 cm	150 cm	818181 cm <sup>3</sup>
B	150 cm	60 cm	90 cm	810,000 cm <sup>3</sup>
C	150 cm	70 cm	80 cm	840,000 cm <sup>3</sup>
* D *	* 150 cm *	* 80 cm *	* 80 cm *	* 960,000 cm <sup>3</sup> *

Final  
solutions

Formula for Volume:  
l x w x h

Work is well labeled  
and clear.

The student displays results  
in a organized format.