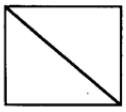
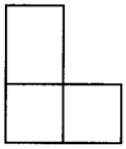
Euler's Dilemma

Try tracing the figures on the following page without lifting your pencil from the paper, without retracing any lines or crossing any lines. Some figures you can trace and some figures you cannot. Euler, an early mathematician, studied figures like these and came up with a rule so that you would know which figures could be traced and which could not.

He found it helpful to categorize vertices as odd and even, depending on the number of line segments coming from each vertex. Investigate figures to see if you can come up with Euler's rule for classifying figures as traceable and those that are not traceable.

1. 

2. 

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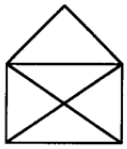
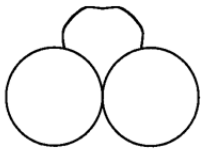
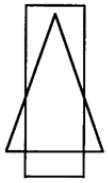
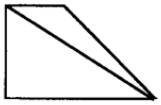
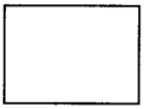
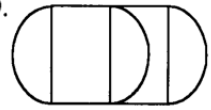
4. 

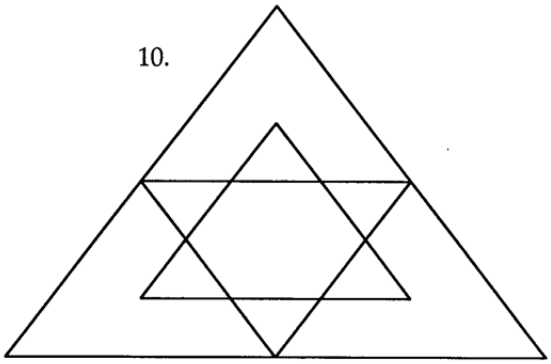
5. 

6. 

7. 

8. 

9. 

10. 

Grade Levels 6 - 8

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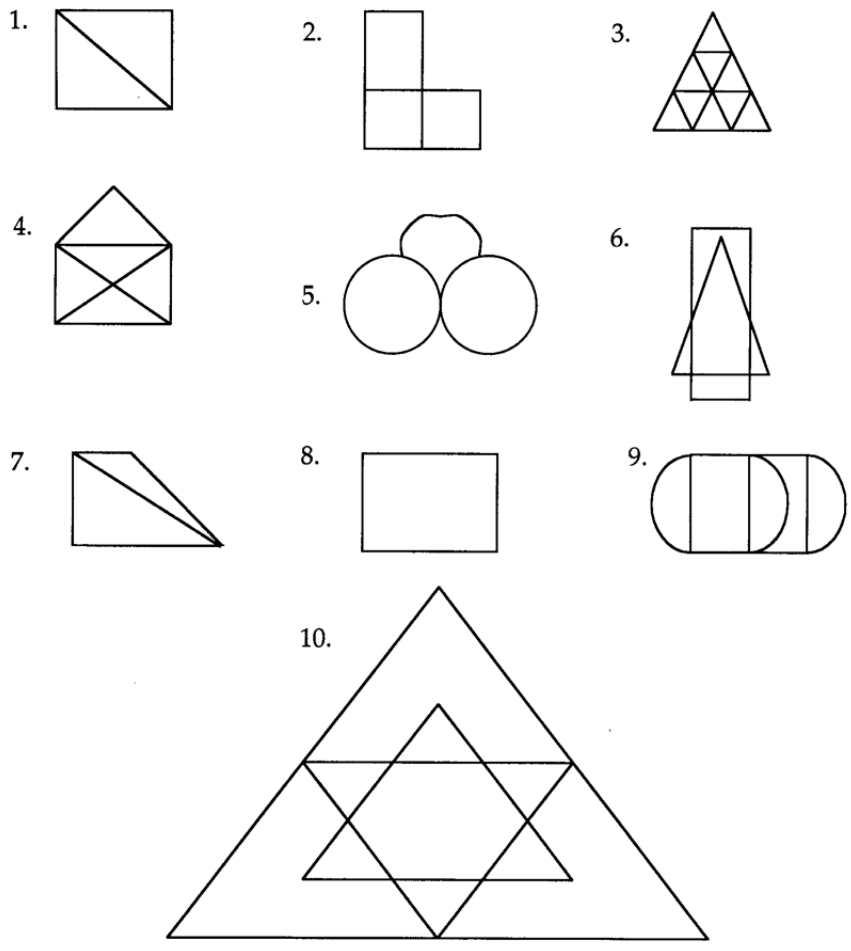

Context

The class was measuring and classifying angles and naming polygons. We were also working on looking for evidence in student work to begin self-assessing problem solving. I had the students thinking of themselves as investigators looking for evidence in their work, as well as other student work. This problem seemed to go along with our investigative work from a different angle. Instead of looking for evidence, they needed to produce the evidence.

What This Task Accomplishes

This task allows me to determine which students can experiment, record results and look for patterns when what they are looking for is obscure. A student will need a way to organize the results of their experimenting carefully so they can begin to see patterns.

What the Student Will Do

All students started to trace the figures I gave them. Most students at this point only recorded whether the figure was traceable or not. Some realized that a chart that kept track of the number of odd and even vertices probably would be helpful since that was clearly important . It was mentioned in the problem and in our discussion of the problem I emphasized the importance of odd and even vertices. As students came up with rules, some tried out their rules on figures they drew and tried to trace.

Time Required for Task

Two, 45-minute class periods

Interdisciplinary Links

This task can be used on units that study famous mathematicians, art projects and puzzles.

Teaching Tips

Be sure the students understand odd and even vertices and that this knowledge is needed to come up with the rule. I let kids (Ha! I really did not have much choice - the investigation lends itself to discussion of the traceable and untraceable figures.) discuss which given figures were traceable and which were not so that everyone had the correct information to work with. If students came up with a rule that was not correct, I often tried to make a figure that would contradict their rule, so that they could go back and look at their data again.

Suggested Materials

- *Extra copies (or tracing paper) of the figures

- Transparencies (with dry erase markers) so kids can trace shapes without altering them.
- Graph paper (Some kids might want to organize their chart.)

*Some kids trace so many times over one figure that they do not have a clean copy to work with.

Possible Solutions

If the figure has zero or two odd vertices, the figure can be traced. You start at one of the odd vertices and end at the other odd vertex.

Benchmark Descriptors

Novice
A Novice paper would be one where the student has evidence of tracing the figures, but really does not see how odd and even vertices help in solving the problem and does not have a strategy that would help make a rule.

Apprentice
An Apprentice paper will have evidence that the student traced the figures correctly and has taken a stab at coming up with the rule. There is evidence that they are thinking about odd and even vertices, but cannot quite come up with a rule that works for all the shapes. They may come up with some rules that work with some figures, but not all figures.

Practitioner
A Practitioner sees that if a figure has zero or two odd vertices it can be traced. They may or may not try some figures of their own to verify their solution. I have also considered work to be a Practitioner level if the student indicates a figure with less than three odd vertices is a solution, even if they do not see that a figure cannot have less than three odd vertices.

Expert
An Expert paper shows that a figure with zero or two odd vertices can be traced. They also show a deeper understanding of odd and even vertices by explaining why this rule holds. They show that they have also tried the rule on other shapes. An Expert may also make mathematically relevant observations that demonstrate higher level thinking skills.

Novice

EULER

This student does not understand the problem. S/he only traced the figures and did not understand the rule. S/he did not consider odd or even.

Im working on a project that you must try and trace over lines without going backward or chosing over other lines.

In the beginning on the first problem I had a little trouble but I looked it over and found a different way to doit and solved the problem. The problem was because I was starting in the wrong place. I was more carful and solved all but two which you could'nt do. I also found that you cant start in the middle of line the is crossing over the problem.

Apprentice

Euler

I thought this was an easy problem of the week but it was really wierd. On some of them I just kept on going over and over them and then I found out I could trace them. The things that I thought were hard were the rules but here are some rules I found.

Some attempts at finding a rule. Rule 1 has some good reasoning and is a good beginning.

1. On all of them except the ones you can't do trace, start at an odd verticy.
2. On all the ones you cant trace. don't have odd or even verticies.

The way I got these rules is I kept going over and over them until I found a rule and once I found one I kept going and going until I found A different one.

Practitioner

Euler Lines

One day Mrs. Fourseth gave us problem of the week. It said exactly this:

Can you draw the design below without crossing or retracing a line or lifting your pencil from the paper?

Investigate tracing figures. Come up with some rules.

I thought that the tracing part would be easy, but the rules would be really tough. I was right. However, we were not completely on our own for the rule because at the bottom of the sheet it said that the rules had something to do with odd and even vertices. What are vertices you asked. They are little dots with an even or odd amount of lines coming out of the dots.

I started out by just trying to solve the shapes on the piece of paper. After I had gotten most of the shapes I decided to try and figure out a rule I looked at all of the shapes that could be done and all of the shapes that could not be done. I counted how many odd and even vertices there were and compared the amounts. I realized that the shapes that you could do had more even vertices then odd. On the Shapes that you couldn't trace there were more odd vertices than even. That rule worked on every shape on the sheet. When Mrs. Fourseth had seen my rule she had drawn a shaped that the rule didn't work on.

After I had seen that my first rule didn't work I had decided to make a graph. I tried to see if I could come up with another rule. Well I didn't come up with a rule, but I did come up with a fact. The fact was if you start on an odd vertice that you would end on an odd vertice, and it was just the same with evens. I know that wasn't really a rule so I decide to go back to the drawing board.

In a couple of days my mother came over and tried to come up with a rule. After about 15 minutes she came up with a rule. It was if the form has at least twice as many odds and evens, then it cannot be traced. When she thought that was a rule I decided to just make sure and I tested it on a shape, but the rule didn't work.

Involving parents in finding solutions could be a problem, depending on how you are going to use the work for assessment. This student is obviously verbal and very comfortable writing about his/her feelings-which is great, but look for good math evidence, reasoning language and representation.

Expert

I then said to myself, well, I'm not getting anywhere today, so why don't I come back to this tomorrow. I went back the next day and I had my dad try to find out a rule with me. He came up with ones that worked on some of them, but not others. Then I was sort of doodling on my paper just thinking about the rule when a little rule popped into my head. If there are more than 2 odd vertices in the shape you cannot trace it. At first it seemed a little bizarre that I had just come up with a rule and even more bizarre since it was so simple. I checked it out on the shapes and sure enough it worked. I decided to make up some and see if it worked on those shapes and it did. I was so happy that I had figured out a rule. I was just hoping that Mrs. Fourseth didn't come up with a shape that it didn't work on!

At first when I came up with a rule and I didn't know that it didn't work I thought that this sort of problem was going to be on the easy side, but I was wrong, because the more work on the problem the more your brain gets worn out.

Good math representation.

My First Graph/Chart

	VERTICES	ODD VERT'S	EVEN VERT'S	START@ODD Y.	START@EVEN Y.	END@ODD Y.	END@EVEN Y.
1	4	2	2	YES		YES	
2	8	2	5	YES		YES	
3	10	0	10		YES		YES
4	6	2	4	YES		YES	
5	4	2	2	YES		YES	
6	11	0	11		YES		YES
7	4	2	2	YES		YES	
8	5	4	1				
9	6	4	2				
10	15	0	15		YES		YES

Expert

Euler

We have recievid a math problem abaut lines and vertices. Euler a Swis mathnatecian discovered a rule about whether or not a fegire can be tracede without lifting your pencil or crossing a line. We learnedabaut odd and even vertices. We are supposed to fenedout some rules about what makes a figure tracable

I started by trying to trace each of the designs that were given to us. After trying all of them I concluded that all of the figures but eight and nine are possible. I looked at eight and nine to see why they were inpossible Both eight and nine have few odd vertices

At first I thought the rule was if there were three or more odd vertices it wouldnit be traced befu then after drawing some figures I realized that if there were odd vertices there had to be an even amount.

Evidence of good reasoning.

Expert

I figured out that when there are two odd vertices you need to start on one of the vertices and end on the other. You need to do this as a vertex will be left with only one line leading in to it after the other lines have been traced.

It is impossible to do the problem if are more than two odd vertices

Good math representation.

The student gives good reasons for having an even number of odd vertices.The student shows deep understanding of the problem.

Problem Number	can't be traced	odd vertices	even vertices	total vertices
1	Y	2	2	3
2	Y	2	6	10
3	Y	0	10	6
4	Y	2	4	4
5	Y	2	2	4
6	Y	0	11	4
7	Y	5	2	6
8	N	4	2	6
9	N	4	2	6
10	Y	0	15	15